HP-33S Calculator Program　　　Closure　　　4

Three-Point Horizontal Resection Reduction Program

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Line	Instruction	Display	User Programming Instructions
R0001	LBL R		
R0002	SF 10		FLAGS SF .0
R0003	ENTER LEFT X		EQN RCL E RCL N etc. ENTER to end
R0004	PSE		
R0005	INPUT X		
R0006	STO A		
R0007	ENTER LEFT Y		EQN RCL E RCL N etc. ENTER to end
R0008	PSE		
R0009	INPUT Y		
R0010	STO B		
R0011	ENTER MID X		EQN RCL E RCL N etc. ENTER to end
R0012	PSE		
R0013	INPUT X		
R0014	STO C		
R0015	ENTER MID Y		EQN RCL E RCL N etc. ENTER to end
R0016	PSE		
R0017	INPUT Y		
R0018	STO D		
R0019	ENTER RIGHT X		EQN RCL E RCL N etc. ENTER to end
R0020	PSE		
R0021	INPUT X		
R0022	STO E		
R0023	ENTER RIGHT Y		EQN RCL E RCL N etc. ENTER to end
R0024	PSE		
R0025	INPUT Y		
R0026	STO F		
R0027	ENTER ALPHA		EQN RCL E RCL N etc. ENTER to end
R0028	PSE		
R0029	INPUT X		
R0030	→HR		
R0031	STO G		
R0032	ENTER BETA		EQN RCL E RCL N etc. ENTER to end
R0033	PSE		
R0034	INPUT X		
R0035	→HR		
R0036	STO H		
R0037	RCL A		
R0038	RCL– C		
R0039	RCL B		

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R0040	RCL– D		
R0041	→ θ, r		↰ 4
R0042	STO L		
R0043	x < > y		
R0044	STO M		
R0045	RCL E		
R0046	RCL– C		
R0047	RCL F		
R0048	RCL– D		
R0049	→ θ, r		↰ 4
R0050	STO K		
R0051	x < > y		
R0052	STO N		
R0053	360		
R0054	STO Z		
R0055	RCL M		
R0056	RCL– N		
R0057	x < 0 ?		
R0058	RCL+ Z		
R0059	STO I		
R0060	RCL+ G		
R0061	RCL+ H		
R0062	RCL Z		
R0063	x < > y		
R0064	–		
R0065	STO S		
R0066	RCL L		
R0067	RCL H		
R0068	SIN		
R0069	×		
R0070	RCL÷ K		
R0071	RCL G		
R0072	SIN		
R0073	÷		
R0074	RCL S		
R0075	SIN		
R0076	÷		
R0077	RCL S		
R0078	TAN		
R0079	1/x		
R0080	+		
R0081	1/x		
R0082	ATAN		
R0083	STO X		
R0084	RCL M		
R0085	180		
R0086	+		

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R0087	RCL+ G		
R0088	RCL+ X		
R0089	STO Y		
R0090	RCL L		
R0091	RCL X		
R0092	SIN		
R0093	×		
R0094	RCL G		
R0095	SIN		
R0096	÷		
R0097	RCL Y		
R0098	x < > y		
R0099	→ *y*, *x*		4
R0100	RCL+ D		
R0101	STO Y		
R0102	x < > y		
R0103	RCL+ C		
R0104	STO X		
R0105	UNKNOWN X =		EQN RCL U RCL N etc. ENTER to end
R0106	PSE		
R0107	VIEW X		
R0108	UNKNOWN Y =		EQN RCL U RCL N etc. ENTER to end
R0109	PSE		
R0110	VIEW Y		
R0111	RCL I		
R0112	RCL+ G		
R0113	RCL+ H		
R0114	→HMS		
R0115	CHECK VALUE		EQN RCL C RCL H etc. ENTER to end
R0116	PSE		
R0117	STOP		
R0118	CF 10		FLAGS CF .0
R0119	RTN		

Notes

(1) Horizontal 3-point resection solution, based on measuring two angles at an unknown point to three known points.

(2) Brief prompts are provided before each requirement for data entry, as well as before results are displayed. The prompt shows for about 1 second, and is then replaced by the value or request for input.

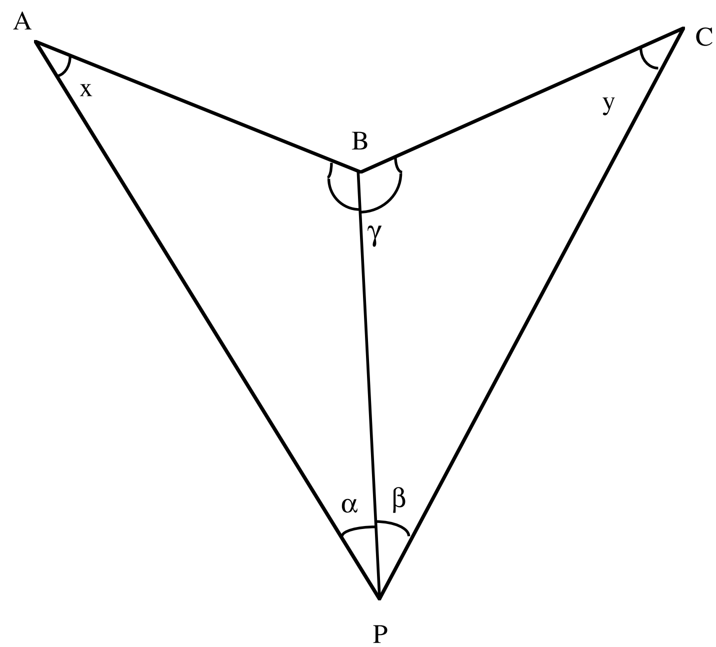
(3) Co-ordinates of the unknown point are displayed following brief prompts. They are also stored in registers for later retrieval.

(4) Angles are entered and displayed in HP notation, i.e., DDD.MMSS. Internal storage of angles and bearings is in decimal degrees.

Three Point Horizontal Resection Reduction Program

Theory

This 2-D resection uses Ormsby's solution. In the discussion below, A is the left point, B is the middle point, C is the right point, and P is the unknown point. The left angle is alpha (α) and the right angle is beta (β). The interior angle at B is gamma (γ). The angle at point A is x, which is the first objective of the solution.



α and β are angles observed from the point P to points A, B and C, whose co-ordinate are known.

$$BP = \frac{AB \sin x}{\sin \alpha} = \frac{BC \sin y}{\sin \beta}$$

$$\text{and } (x + y) = (360° - (\alpha + \beta + \gamma)) = s$$

$$\frac{AB}{\sin \alpha} \sin x = \frac{BC}{\sin \beta} \sin (s - x) = \frac{BC}{\sin \beta}(\sin s \cos x - \cos s \sin x)$$

$$\frac{AB}{\sin \alpha} \sin x = \frac{BC}{\sin \beta} \sin s \cos x - \frac{BC}{\sin \beta} \cos s \sin x$$

$$\sin x \left(\frac{AB}{\sin \alpha} + \frac{BC}{\sin \beta} \cos s\right) = \frac{BC}{\sin \beta} \sin s \cos x$$

$$\left(\frac{AB}{\sin \alpha} + \frac{BC}{\sin \beta} \cos s\right) \frac{\sin \beta}{BC \sin s} = \cot x$$

$$\frac{AB \sin\beta}{BC \sin \alpha \sin s} + \frac{BC \cos s \sin \beta}{BC \sin s \sin \beta} = \cot x$$

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$$\frac{AB \sin \beta}{BC \sin \alpha \sin s} + \cot s = \cot x$$ [this is the equation solved first]

$$y = s - x$$

With x and y determined, the sides AP, BP and CP can be calculated and hence the co-ordinates of P, as follows:

The azimuth of BP (Az_{BP}) can be determined using $Az_{BP} = Az_{AB} + \alpha + x$

The length of BP can be determined using $BP = \frac{AB \sin x}{\sin \alpha}$

Knowing the co-ordinates of B, Az_{BP} and BP, the co-ordinates of P can be easily computed. As a check, the equivalent solution can be obtain through the sides AP or CP, or using the angle y. Note that if P is close the danger circle, a solution will still be obtained, but the sum of $\alpha + \beta + \gamma$ will be close to 180°, probably in the range 175° to 185°. In this case, the solution will be highly sensitive to changes in α and β. If the solution is close to the danger circle, recomputed with the angles changed by about their precision and see how much the resulting co-ordinates change. It can be quite surprising!

Whole circle bearings in HP notation are used. Arbitrary co-ordinates are satisfactory. Plane surveying assumptions apply. The program uses no error checking on entered data. A check is made by showing the sum $\alpha + \beta + \gamma$. If this is close to 180°, the unknown point lies close to the danger circle and the result is highly suspect.

Sample Computation

Known Points

Point Name	X	Y
Point A	–25.336	778.136
Point B	–27.465	1179.927
Point C	–30.297	1555.643

Angles Left (α) = 136° 35' 26"

Right (β) = 27° 19' 24"

Results Unknown Point (P) X Co-ordinate = 26.009

Unknown Point (P) Y Co-ordinate = 1101.818

Check Angle = 344° 02' 32"

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Storage Registers Used

A Left known point – X co-ordinate

B Left known point – Y co-ordinate

C Middle known point – X co-ordinate

D Middle known point – Y co-ordinate

E Right known point – X co-ordinate

F Right known point – Y co-ordinate

G Left measured angle — alpha (α)

H Right measured angle — beta (β)

I Interior angle at Middle known point — gamma (γ)

K Distance middle to right point

L Distance middle to left point

M Bearing of middle to left point in decimal degrees

N Bearing of middle to right point in decimal degrees

S s = x + y in decimal degrees

X Initial inputs, then angle x, then X co-ordinate of unknown point

Y Initial inputs, then bearing from middle to unknown point, then Y co-ordinate of unknown point

Z 360

Labels Used

Label **R** Length = 503 Checksum = 212C

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

Running the Program

Press XEQ R

Prompt ENTER LEFT X briefly, then X?

Enter X Xo-ordinate for left known point.

Press R/S.

Prompt ENTER LEFT Y briefly, then Y?

Enter Y Xo-ordinate for left known point.

Press R/S.

Prompt ENTER MID X briefly, then X?

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Enter X Xo-ordinate for middle known point.

Press R/S.

Prompt ENTER MID Y briefly, then Y?

Enter Y Xo-ordinate for middle known point.

Press R/S.

Prompt ENTER RIGHT X briefly, then X?

Enter X Xo-ordinate for right known point.

Press R/S.

Prompt ENTER RIGHT Y briefly, then Y?

Enter Y Xo-ordinate for right known point.

Press R/S.

Prompt ENTER ALPHA briefly, then X?

Enter left angle (α) in HP notation.

Press R/S.

Prompt ENTER BETA briefly, then X?

Enter right angle (β) in HP notation.

Press R/S.

RUNNING……

Prompt UNKNOWN X briefly, then X=

X co-ordinate of unknown point (P) is displayed.

Press R/S.

Prompt UNKNOWN Y briefly, then Y=

Y co-ordinate of unknown point (P) is displayed.

Press R/S.

Prompt CHECK VALUE briefly.

Sum $\alpha + \beta + \gamma$ is displayed in lower line of display in HP notation.

Check that value is not too close to 180°. At least 5 ° away, preferably 15* or more away.

Press R/S to clear flags. Program ends.

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Sample Computation 2

Known Points

Point Name	X	Y
Point A	133.639	1548.712
Point B	158.065	1492.276
Point C	150.267	1353.056

Angles

Left (α) = 5° 01' 48"

Right (β) = 3° 41' 29"

Results

Unknown Point (P) X Co-ordinate = 116.784

Unknown Point (P) Y Co-ordinate = 1186.818

Check Angle = 162° 06' 44"

This is not the ideal arrangement for a resection, as the measured angles are quite small. But the program will still produce an acceptable result.

This example is provided because the other example has negative co-ordinates and this tends to increase the chances of incorrect data entry. It happened to me, twice!