HP-33S Calculator Program Closure 8

Crandall's Adjustment for a Closed Traverse

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Line	Instruction	Display	User Instructions
K0001	LBL K		
K0002	CL Σ		
K0003	SF 10		(Use FLAGS SF .0)
K0004	ENTER FIX SIDES		(Use EQN RCL E, RCL N, RCL T, etc.)
K0005	PSE		
F0001	LBL F		
F0002	ENTER F AZ		(Use EQN RCL E, RCL N, RCL T, etc.)
F0003	PSE		
F0004	INPUT Q		
F0005	RCL Q		
F0006	x < 0?		
F0007	GTO V		
F0008	RCL Q		
F0009	→HR		
F0010	STO Q		
F0011	ENTER F DIST		(Use EQN RCL E, RCL N, RCL T, etc.)
F0012	PSE		
F0013	INPUT D		
F0014	RCL Q		
F0015	RCL D		
F0016	θ,r → y,x		
F0017	Σ+		
F0018	GTO F		
V0001	LBL V		
V0002	31		
V0003	STO *i*		*i* is on decimal point key.
V0004	0		
V0005	STO (*i*)		(*i*) is on ENTER key.
V0006	32		
V0007	STO *i*		
V0008	0		
V0009	STO (*i*)		
V0010	33		
V0011	STO *i*		
V0012	0		
V0013	STO (*i*)		
V0014	ENTER VAR SIDES		(Use EQN RCL E, RCL N, RCL T, etc.)
V0015	PSE		
U0001	LBL U		

Crandall's Adjustment for a Closed Traverse

Line	Instruction	Display	User Instructions
U0002	ENTER V AZ		(Use EQN RCL E, RCL N, RCL T, etc.)
U0003	PSE		
U0004	INPUT Q		
U0005	RCL Q		
U0006	x < 0?		
U0007	GTO J		
U0008	RCL Q		
U0009	→HR		
U0010	STO Q		
U0011	ENTER V DIST		(Use EQN RCL E, RCL N, RCL T, etc.)
U0012	PSE		
U0013	INPUT D		
U0014	RCL Q		
U0015	RCL D		
U0016	θ,r → y,x		
U0017	Σ+		
U0018	GTO U		
J0001	LBL J		
J0002	Σx^2		(Enter using SUMS Σx^2)
J0003	Σy^2		(Enter using SUMS Σy^2)
J0004	×		
J0005	Σxy		(Enter using SUMS Σxy)
J0006	x^2		
J0007	–		
J0008	STO D		
J0009	Σy		(Enter using SUMS Σy)
J0010	Σxy		(Enter using SUMS Σxy)
J0011	×		
J0012	Σx		(Enter using SUMS Σx)
J0013	Σy^2		(Enter using SUMS Σy^2)
J0014	×		
J0015	–		
J0016	RCL÷ D		
J0017	STO A		
J0018	Σx		(Enter using SUMS Σx)
J0019	Σxy		(Enter using SUMS Σxy)
J0020	×		
J0021	Σy		(Enter using SUMS Σy)
J0022	Σx^2		(Enter using SUMS Σx^2)
J0023	×		
J0024	–		
J0025	RCL÷ D		
J0026	STO B		
J0027	0		
J0028	STO V		
J0029	ENTER VAR SIDES		(Use EQN RCL E, RCL N, RCL T, etc.)

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Crandall's Adjustment for a Closed Traverse

Line	Instruction	Display	User Instructions
J0030	PSE		
L0001	LBL L		
L0002	ENTER V AZ		(Use EQN RCL E, RCL N, RCL T, etc.)
L0003	PSE		
L0004	INPUT Q		
L0005	RCL Q		
L0006	x < 0?		
L0007	GTO M		
L0008	RCL Q		
L0009	→HR		
L0010	STO Q		
L0011	ENTER V DIST		(Use EQN RCL E, RCL N, RCL T, etc.)
L0012	PSE		
L0013	INPUT D		
L0014	RCL Q		
L0015	RCL D		
L0016	θ,r → y,x		
L0017	RCL× D		
L0018	RCL× A		
L0019	x < > y		
L0020	RCL× D		
L0021	RCL× B		
L0022	+		
L0023	STO C		
L0024	STO+ V		
L0025	DIST CORRN		(Use EQN RCL D, RCL I, RCL S, etc.)
L0026	PSE		
L0027	VIEW C		
L0028	RCL C		
L0029	STO+ D		
L0030	CORRD DIST		(Use EQN RCL C, RCL O, RCL R, etc.)
L0031	PSE		
L0032	VIEW D		
L0033	GTO L		
M0001	LBL M		
M0002	SUM OF CORRNS		(Use EQN RCL S, RCL U, etc.)
M0003	PSE		
M0004	VIEW V		
M0005	CF 10		(Use FLAGS CF .0)
M0006	RTN		

Notes

(1) This program uses a lot of labels (7), because of the need to loop through side entry and to work through several choices. Consequently, it should be entered only if it is really needed.

(2) The program can work with traverses where all the sides are to be adjusted, where some of the sides are considered 'fixed' (not to be adjusted), and traverses between known points with co-ordinates. The last need the azimuth and distance between the fixed points derived from the co-ordinates, and treated as a fixed side. (Use the Utility 6 program or Utility 4 key sequence, if needed.)

(3) The sequence of operations is to enter the fixed sides (those not to be adjusted), then the variable sides (those to be adjusted), and then the variable sides again. The reason for entering the variable sides twice is that the calculator does not have sufficient memory to store the sides for processing a second time, so they must be re-entered.

(4) Provided that all the fixed sides are entered first, then all the variable sides, then all the variable sides again, it does not matter in which order the sides within each group are entered.

(5) Azimuths are entered in HP notation, i.e., DDD.MMSS

(6) The misclosure components of the unadjusted traverse in X (or E) and Y (or N) can be displayed by recalling Σy and Σx using the SUMS menu. (Note these are back-to-front.) Reversing their order using x<>y sets them up for conversion to polar. To check the traverse closes after adjustment, enter the sides through one of the other traverse programs, e.g., Closure 1 or Closure 2.

(7) This program was inspired by the program developed by Neil H. Bradbury, LS, in January, 1977, for the HP-25 calculator.

(8) The program assumes that the error in the length of each side is proportional to the length of the side, which is the situation with EDM. It will produce somewhat different results to a Crandall's adjustment based on the error increasing as the square root of the distance, which is the more traditional method based on error propagation using a tape.

(9) To indicate the end of the sides of the traverse to be entered at any point, enter a negative value for the azimuth, –1 is easiest.

Theory

Crandall's Adjustment was developed by Charles Crandall, and published in 1914. It was the first serious rival to Bowditch's Method of adjustment. The fundamental concepts were that angle measurement was far better than distance measurement, and that the small random errors in the angles of a traverse could be properly adjusted by simple distribution of the angular misclosure among the traverse's angles. This meant that all the remaining misclosure in the traverse, i.e., all the random errors, could be accounted for by the distances alone, so these were all that was adjusted by Crandall's Adjustment.

Crandall used least squares adjustment by condition equations for the adjustment. By avoiding having to adjust the angles as part of the adjustment proper, the number of conditions was reduced to two, which greatly simplified the calculations.

Angle Adjustment

The angular misclosure of the traverse can be adjusted by any reasonable means. The misclosure can be distributed evenly among all the angles, or larger amounts may be apportioned to angles that can be assumed to be less reliable, perhaps because of short lines or poor lies of sight. However the angles are adjusted, they should be adjusted to bring the misclosure to zero, and then the azimuths of each line calculated.

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Crandall's Adjustment for a Closed Traverse

It is a good idea to run the traverse through one of the closure programs to make sure that there are no gross or systematic errors in the traverse, prior to adjustment. This is because the adjustment process assumes that only random errors are present in the data. If this assumption is not correct, the adjustment will produce erroneous results.

Distance Adjustment

With the angular misclosure adjusted to zero, there are now only two conditions that must be met to have a consistent traverse. (The third condition, that the angular misclosure is zero, has been met by adjusting the angle ahead of this point.) Solution of a 2×2 matrix is straightforward, so a full least squares adjustment by condition equations can be performed. This adjustment changes only the lengths of the sides of the traverse. The angles are already adjusted and so held fixed.

If the length of each line in the traverse is M_i, while θ_i is its azimuth, then d_i will be the adjusted length of each line in the traverse, and v_i will be the correction to the measured length of each line to obtain the adjusted length:

$$d_i = M_i + v_i \qquad [1]$$

The conditions to be met can be expressed as:

$$d_1 \cos \theta_1 + d_2 \cos \theta_2 + d_3 \cos \theta_3 + \dots = 0 \qquad \text{[Sum of latitudes equals zero]}$$

$$d_1 \sin \theta_1 + d_2 \sin \theta_2 + d_3 \sin \theta_3 + \dots = 0 \qquad \text{[Sum of departures equals zero]}$$

Substituting Equation [1] into the above equations gives:

$$(M_1 + v_1) \cos \theta_1 + (M_2 + v_2) \cos \theta_2 + (M_3 + v_3) \cos \theta_3 + \dots = 0$$

$$(M_1 + v_1) \sin \theta_1 + (M_2 + v_2) \sin \theta_2 + (M_3 + v_3) \sin \theta_3 + \dots = 0$$

Re-arranging and gathering terms produces:

$$v_1 \cos \theta_1 + v_2 \cos \theta_2 + v_3 \cos \theta_3 + \dots + \mathrm{q}_1 = 0$$

$$v_1 \sin \theta_1 + v_2 \sin \theta_2 + v_3 \sin \theta_3 + \dots + \mathrm{q}_2 = 0 \qquad [2]$$

where $\mathrm{q}_1 = M_1 \cos \theta_1 + M_2 \cos \theta_2 + M_3 \cos \theta_3 + \dots =$ misclosure in latitude

$\mathrm{q}_2 = M_1 \sin \theta_1 + M_2 \sin \theta_2 + M_3 \sin \theta_3 + \dots =$ misclosure in departure

So q_1 and q_2 are obtained directly from the closure check of the traverse, done after the angles are adjusted and azimuths computed. Representing the departure of each line by D_i, and the latitude of each line by L_i, Equations [2] can be rewritten as:

$$v_1 \cos \frac{L_1}{d_1} + v_2 \cos \frac{L_2}{d_2} + v_3 \cos \frac{L_3}{d_3} + \dots + \mathrm{q}_1 = 0$$

$$v_1 \sin \frac{D_1}{d_1} + v_2 \sin \frac{D_2}{d_2} + v_3 \sin \frac{D_3}{d_3} + \dots + \mathrm{q}_2 = 0 \qquad [3]$$

The theory of least squares now allows the development of what are termed the normal equations (named after the normal distribution, which describes the distribution characteristics of the random errors). The details can be found elsewhere, but basically they are a combination of all the measured data, the precision values of the measurements, and the models that describe how they all fit together, i.e., the model of the conditions to be met (e.g., Equations [2] and [3]).

There are two cases that Crandall considered. The first is the situation where the error in the length of the sides increases in proportion to the square root of the length of the line, i.e., $\sqrt{d}$, which is a good assumption for taped distances. The second case is where the error in distance increases in proportion to the length of the line, which is more the case with short distances measured with rods, but also the case with EDM. This program is based on the second case, as it is more probable that EDM traverses will be used with this program.

If the error in the lengths of the sides of the traverse increases in proportion to the length of each side, then the normal equations for the adjustment are as follows:

$$A\sum_i L_i^2 + B\sum_i L_i D_i + q_1 = 0$$

$$A\sum_i L_i D_i + B\sum_i D_i^2 + q_2 = 0 \qquad [4]$$

Solving for A and B, which are called the correlatives:

$$\mathrm{A} = \frac{q_2\sum_i L_i D_i - q_1\sum_i D_i^2}{\sum_i D_i^2 \sum_i L_i^2 - \left(\sum_i L_i D_i\right)^2}$$

$$\mathrm{B} = \frac{q_1\sum_i L_i D_i - q_2\sum_i L_i^2}{\sum_i D_i^2 \sum_i L_i^2 - \left(\sum_i L_i D_i\right)^2} \qquad [5]$$

the corrections to the line lengths can be calculated using:

$$v_i = L_i \ d_i \ \mathrm{A} + D_i \ d_i \ \mathrm{B} \qquad [6]$$

As an aside, the corrections should all sum to zero if the traverse has no fixed sides, but that can be affected by rounding during the calculations, especially if the sums of the various components are large, so the result may be not quite zero. If the traverse has fixed sides, the sum of the corrections will be significantly different from zero depending upon the length of the fixed side.

Including Fixed Sides and Points

In the event that the traverse includes more than one 'fixed' point (usually the starting point in calculations), Crandall's Method can still be used. A 'fixed' point means a point that has known co-ordinates, usually from a more precise prior survey, and will not be subject to adjustment in this program. This situation arises when a traverse connects to more than one known point.

Crandall's Adjustment for a Closed Traverse

If there are two fixed points, and these form the end points of one line of the traverse, then the traverse calculations are done in the normal way, except that the L_i, D_i, and L_iD_i values for the fixed line are not calculated and not included in the summations to solve the correlatives. This is the case in this program.

If there are more than two fixed points, or the fixed points are not adjacent, the traverse must be split into sections between fixed points, and the sections adjusted independently. This is not the ideal solution, as a full least squares adjustment should be used in this situation.

If the traverse runs between two known points, then the line (vector) between the two known points (computed from the co-ordinates of the two points) can be considered another side in the traverse. Computing around the traverse, the misclosure will reflect how well the traverse agrees with this computed vector. To adjust the traverse, don't include the L_i, D_i, and L_iD_i values for this line in the summations, and all the corrections will occur in the measured traverse lines.

For this program, any fixed lines should be worked out ahead of time, including the line between known end-points of the traverse (if applicable). The first part of the program allows the user to enter these sides, and they do not add to the data that sets up the adjustment. The second part of the program allows the user to enter the sides that do add to the adjustment, and these are entered again for the actual adjustment process.

Sample Computations and Running the Program

1. All sides to be adjusted

The traverse to be adjusted is as follows:

Azimuth	Distance
0° 12'	156.41
90° 00'	211.65
165° 49'	173.82
250° 55'	176.60
308° 30'	112.26

The initial misclosure is: DE = 0.036

DN = 0.033

Misclosure Length = 0.049

Misclosure Azimuth = 47° 24' 15"

Running the Program

Press XEQ K to start the program.

The program shows ENTER FIX SIDE to indicate that the fixed sides should be entered first.

The program then prompts ENTER F AZ, to indicate that the azimuth of a fixed side should be entered in DDD.MMSS (HP) notation. The prompt is to enter Q?.

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Crandall's Adjustment for a Closed Traverse

As there are no fixed sides in this case, key in 1 then press the +/– key, and press R/S.

The calculator shows ENTER VAR SIDE to indicate that the sides to be varied (adjusted) should be entered now.

The calculator then prompts ENTER V AZ to indicate that the azimuth of a variable sides should be entered in DDD.MMSS (HP) notation. The prompts to enter is Q?.

Key in the azimuth of the first side, 0.12, then press R/S.

The calculator prompts with ENTER V DIST, to indicate that the length of that variable side should be entered. The prompt is to enter D?.

Key in the distance 156.41, then press R/S.

The calculator prompts with ENTER V AZ, then the Q? prompt.

Key in the next azimuth, 90.0, and press R/S.

The calculator prompts with ENTER V DIST, then the D? prompt.

Key in the distance, 211.65, and press R/S.

The calculator prompts with ENTER V AZ, then the Q? prompt.

Key in the next azimuth, 165.49, and press R/S.

The calculator prompts with ENTER V DIST, then the D? prompt.

Key in the distance, 173.83, and press R/S.

The calculator prompts with ENTER V AZ, then the Q? prompt.

Key in the next azimuth, 250.55, and press R/S.

The calculator prompts with ENTER V DIST, then the D? prompt.

Key in the distance, 176.6, and press R/S.

The calculator prompts with ENTER V AZ, then the Q? prompt.

Key in the next azimuth, 308.3, and press R/S.

The calculator prompts with ENTER V DIST, then the D? prompt.

Key in the distance, 112.26, and press R/S.

The calculator prompts with ENTER V AZ, then the Q? prompt.

As this is the last variable side, key in a negative azimuth, –1, and press R/S.

The calculator prompts with ENTER VAR SIDE, and prepares to receive the variable sides again for the adjustment calculation.

The calculator prompts with ENTER V AZ, then the Q? prompt.

Key in the azimuth of the first side, 0.12, then press R/S.

The calculator prompts with ENTER V DIST, then the D? prompt.

Key in the length of the first side, 156.41, then press R/S.

The calculator shows DIST CORRN, indicating the correction to the length of that side, then shows the correction, using the C= prompt. In this case, it is –0.0139. Press R/S.

Crandall's Adjustment for a Closed Traverse

The calculator shows CORRD DIST, indicating the corrected (adjusted) distance of the side, then shows the corrected distance, using the D= prompt. In this case, it is 156.3961. Press R/S.

The calculator prompts with ENTER V AZ, then the Q? prompt.

Key in the azimuth of the next side, 90, then press R/S.

The calculator prompts with ENTER V DIST, then the D? prompt.

Key in the length of the side, 211.65, then press R/S.

The calculator shows DIST CORRN, indicating the correction to the length of that side, then shows the correction, using the C= prompt. In this case, it is –0.0207. Press R/S.

The calculator shows CORRD DIST, indicating the corrected (adjusted) distance of the side, then shows the corrected distance, using the D= prompt. In this case, it is 211.6293. Press R/S.

The calculator prompts with ENTER V AZ, then the Q? prompt.

Key in the azimuth of the next side, 165.49, then press R/S.

The calculator prompts with ENTER V DIST, then the D? prompt.

Key in the length of the side, 173.82, then press R/S.

The calculator shows DIST CORRN, indicating the correction to the length of that side, then shows the correction, using the C= prompt. In this case, it is 0.0132. Press R/S.

The calculator shows CORRD DIST, indicating the corrected (adjusted) distance of the side, then shows the corrected distance, using the D= prompt. In this case, it is 173.8332. Press R/S.

The calculator prompts with ENTER V AZ, then the Q? prompt.

Key in the azimuth of the next side, 250.55, then press R/S.

The calculator prompts with ENTER V DIST, then the D? prompt.

Key in the length of the side, 176.6, then press R/S.

The calculator shows DIST CORRN, indicating the correction to the length of that side, then shows the correction, using the C= prompt. In this case, it is 0.0194. Press R/S.

The calculator shows CORRD DIST, indicating the corrected (adjusted) distance of the side, then shows the corrected distance, using the D= prompt. In this case, it is 176.6194. Press R/S.

The calculator prompts with ENTER V AZ, then the Q? prompt.

Key in the azimuth of the last side, 308.3, then press R/S.

The calculator prompts with ENTER V DIST, then the D? prompt.

Key in the length of the last side, 112.26, then press R/S.

The calculator shows DIST CORRN, indicating the correction to the length of that side, then shows the correction, using the C= prompt. In this case, it is 0.0001. Press R/S.

The calculator shows CORRD DIST, indicating the corrected (adjusted) distance of the side, then shows the corrected distance, using the D= prompt. In this case, it is 112.2601. Press R/S.

The calculator prompts with ENTER V AZ, then the Q? prompt.

As this is the last side, key in a negative azimuth, –1, then press R/S.

The calculator shows SUM OF CORRNS, then shows the sum of the corrections to the distances, using the V= prompt. In this case, the sum is –0.0019, which is reasonably close to zero.

Crandall's Adjustment for a Closed Traverse

Pressing R/S again, resets the flags and ends the program.

Note that the sides in each group may be entered in any order. When they variable sides are entered a second time to calculate the corrections to their lengths, sides may be entered more than once, e.g., to double-check the results, but this will lead to errors in the sum of the corrections value displayed at the end of the program.

Summarizing the results, the traverse is now adjusted to be:

Azimuth	Adjusted Distance
0° 12'	156.3961
90° 00'	211.6293
165° 49'	173.8332
250° 55'	176.6194
308° 30'	112.2601

The final misclosure is: DE = –0.000 015

DN = 0.000 040

Misclosure Length = 0.000 042

Misclosure Azimuth = 339° 35'

The misclosure is now smaller than the finest distance measurement, so all that remains is rounding error in the calculations.

2. One fixed side, a traverse between known points

For this traverse, one side is considered fixed, because the traverse is between two fixed points. The azimuth and distance between the two points are deduced from co-ordinates (using a different program), and are 59° 04' 33" for 1995.78. The angular misclosure is adjusted out of the traverse angles, and the azimuths of the sides calculated, on the same azimuth datum as the fixed side, i.e., the same co-ordinate system.

The program is started and the fixed side entered: 59.0433 and 1995.78.

As this is the only fixed side, the next azimuth is entered as –1, and the calculator moves to collect the data for the variable sides. These are as follows:

Azimuth	Distance
349° 47' 19"	429.20
109° 22' 12"	476.00
202° 58' 13"	1362.00
271° 07' 59"	210.10
259° 42' 45"	164.00
295° 34' 53"	319.80
237° 45' 39"	499.95
283° 57' 32"	482.10

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Crandall's Adjustment for a Closed Traverse

The initial misclosure is:	DE	=	2.910
	DN	=	–1.290
	Misclosure Length	=	3.183
	Misclosure Azimuth	=	113° 54' 23"

These are entered into the calculator as prompted. Once they have all been entered, an azimuth of –1 is entered, then the sides are entered again, for the adjustment results to be calculated. The adjusted sides are:

Azimuth	Distance Correction	Distance
349° 47' 19"	0.4683	429.668
109° 22' 12"	–0.9666	475.033
202° 58' 13"	–0.4317	1361.568
271° 07' 59"	0.1719	210.272
259° 42' 45"	0.0930	164.093
295° 34' 53"	0.4393	320.239
237° 45' 39"	0.5639	500.514
283° 57' 32"	0.9761	483.076

Entering –1 as the next azimuth, the calculator displays 1.314 as the sum of the corrections.

The final misclosure is:	DE	=	0.000 081
	DN	=	–0.000 115
	Misclosure Length	=	0.000 141
	Misclosure Azimuth	=	144° 53'

The misclosure is now smaller than the finest level of distance measurement.

Comments

An important point to note about Crandall's Adjustment is that because all the corrections are placed in the distances, any non-random errors (e.g., gross errors) in the angles will produce very large corrections in the distances. This is an indication that there may be gross errors in the angles, and this should be checked.

In the second example, the original lengths were in many cases almost approximate in their apparent precision, while the misclosure was quite large, being about 1: 1,900. This led to the large corrections in the distances seen in the results.

Note also that in the second example, with the fixed side, the sum of the corrections was 1.314, well away from the zero. This is characteristic of adjusting traverses with fixed sides.

Crandall's Adjustment for a Closed Traverse

Storage Registers Used

A	Correlative A
B	Correlative B
C	Correction to side length
D	Current entered side length, intermediate result in adjustment calculation, adjusted length
Q	Current entered side azimuth
V	Sum of corrections to the lengths
i	Used for indirect addressing of the statistical registers

Statistical Registers:

Σx = Sum of the latitudes

Σy = Sum of the departures

Σx^2 = Sum of the squares of the latitudes of the sides to be adjusted

Σy^2 = Sum of the squares of the departures of the sides to be adjusted

Σxy = Sum of the products of the latitudes and departures of the sides to be adjusted

n = Number of sides entered from start

Labels Used

Label **K**	Length = 30	Checksum = 2762
Label **F**	Length = 76	Checksum = 10B7
Label **V**	Length = 132	Checksum = F531
Label **U**	Length = 76	Checksum = DBF0
Label **J**	Length = 117	Checksum = 22E0
Label **L**	Length = 141	Checksum = 6938
Label **M**	Length = 31	Checksum = 0B96

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computations to check proper operation after entry.

Reference

CRANDALL, Charles L., 1914. *Geodesy and Least Squares*. New York : John Wiley and Sons.

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