Convert Latitude and Longitude to Transverse Mercator Co-ordinates (UTM, SPCS, etc.)

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Line	Instruction	Display	User Instructions
T0001	LBL T		LBL P
T0002	SF 10		FLAGS 1 .0
T0003	LAT–LONG 2 TM		EQN RCL L, RCL A, etc.
T0004	PSE		PSE
T0005	CL x		CLEAR 1
T0006	STO F		
T0007	STO L		
T0008	6378137		a value for ellipsoid (WGS84)
T0009	STO A		
T0010	0.00669438		e^2 value for ellipsoid (WGS84)
T0011	STO E		
T0012	0.9996		k_0 value for zone (UTM 17)
T0013	STO K		
T0014	–81		λ_0 for zone (UTM 17)
T0015	STO B		
T0016	0		ϕ_0 for zone (UTM 17)
T0017	STO C		
T0018	500000		E_0 for zone (UTM 17)
T0019	STO J		
T0020	0		N_0 for zone (UTM 17)
T0021	STO I		
T0022	CHECK–ENTER A		EQN RCL C, RCL H, etc.
T0023	PSE		PSE
T0024	INPUT A		INPUT A
T0025	CHECK–ENTER E		EQN RCL C, RCL H, etc.
T0026	PSE		PSE
T0027	INPUT E		INPUT E
T0028	CHECK–ENTER K		EQN RCL C, RCL H, etc.
T0029	PSE		PSE
T0030	INPUT K		INPUT K
T0031	CHK–NTR LONG 0		EQN RCL C, RCL H, etc.
T0032	PSE		PSE
T0033	INPUT B		INPUT B
T0034	CHK–NTR LAT 0		EQN RCL C, RCL H, etc.
T0035	PSE		PSE
T0036	INPUT C		INPUT C
T0037	CHK–NTR N 0		EQN RCL C, RCL H, etc.

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Line	Instruction
T0038	PSE
T0039	INPUT I
T0040	CHK–NTR E 0
T0041	PSE
T0042	INPUT J
T0043	ENTER LAT
T0044	PSE
T0045	INPUT F
T0046	ENTER LONG
T0047	PSE
T0048	INPUT L
T0049	RCL F
T0050	→HR
T0051	STO F
T0052	RCL L
T0053	→HR
T0054	STO L
T0055	RCL B
T0056	→HR
T0057	STO B
T0058	RCL C
T0059	→HR
T0060	STO C
T0061	1
T0062	RCL F
T0063	SIN
T0064	x^2
T0065	RCL× E
T0066	—
T0067	√x
T0068	RCL A
T0069	x < > y
T0070	÷
T0071	STO N
T0072	1
T0073	RCL— E
T0074	RCL× A
T0075	RCL F
T0076	SIN
T0077	x^2
T0078	RCL× E
T0079	1

Line	Instruction
T0080	x < > y
T0081	—
T0082	1.5
T0083	y^x
T0084	÷
T0085	STO R
T0086	RCL N
T0087	RCL÷ R
T0088	STO P
T0089	RCL F
T0090	TAN
T0091	STO T
T0092	RCL L
T0093	RCL— B
T0094	→RAD
T0095	STO W
T0096	1
T0097	RCL E
T0098	4
T0099	÷
T0100	—
T0101	RCL E
T0102	x^2
T0103	0.046875
T0104	×
T0105	—
T0106	RCL E
T0107	x^3 †
T0108	0.01953125
T0109	×
T0110	—
T0111	STO D
T0112	RCL F
T0113	→RAD
T0114	×
T0115	STO M
T0116	RCL C
T0117	→RAD
T0118	RCL× D
T0119	STO H
T0120	RCL E
T0121	RCL E

Line	Instruction
T0122	x^2
T0123	4
T0124	÷
T0125	+
T0126	RCL E
T0127	x^3 †
T0128	0.1171875
T0129	×
T0130	+
T0131	0.375
T0132	×
T0133	STO D
T0134	RCL F
T0135	2
T0136	×
T0137	SIN
T0138	×
T0139	STO— M
T0140	RCL C
T0141	2
T0142	×
T0143	SIN
T0144	RCL× D
T0145	STO— H
T0146	RCL E
T0147	x^2
T0148	RCL E
T0149	x^3 †
T0150	0.75
T0151	×
T0152	+
T0153	0.05859375
T0154	×
T0155	STO D
T0156	RCL F
T0157	4
T0158	×
T0159	SIN
T0160	×
T0161	STO+ M
T0162	RCL C
T0163	4

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Line	Instruction
T0164	×
T0165	SIN
T0166	RCL× D
T0167	STO+ H
T0168	RCL E
T0169	x^3 †
T0170	35
T0171	×
T0172	3072
T0173	÷
T0174	STO D
T0175	RCL F
T0176	6
T0177	×
T0178	SIN
T0179	×
T0180	STO— M
T0181	RCL A
T0182	STO× M
T0183	RCL C
T0184	6
T0185	×
T0186	SIN
T0187	RCL× D
T0188	STO— H
T0189	RCL A
T0190	STO× H
****	Compute E
T0191	RCL N
T0192	RCL× W
T0193	RCL F
T0194	COS
T0195	×
T0196	STO X
T0197	RCL F
T0198	COS
T0199	RCL× W
T0200	x^3 †
T0201	RCL× N
T0202	6
T0203	÷
T0204	RCL P
T0205	RCL T
T0206	x^2
T0207	—

Line	Instruction
T0208	×
T0209	STO+ X
T0210	1
T0211	RCL T
T0212	x^2
T0213	6
T0214	×
T0215	—
T0216	4
T0217	×
T0218	RCL P
T0219	x^3 †
T0220	×
T0221	RCL T
T0222	x^2
T0223	8
T0224	×
T0225	1
T0226	+
T0227	RCL P
T0228	x^2
T0229	×
T0230	+
T0231	RCL T
T0232	x^2
T0233	2
T0234	×
T0235	RCL× P
T0236	—
T0237	RCL T
T0238	4
T0239	y^x
T0240	+
T0241	RCL× N
T0242	RCL F
T0243	COS
T0244	RCL× W
T0245	5
T0246	y^x
T0247	×
T0248	120
T0249	÷
T0250	STO+ X
T0251	61
T0252	RCL T

Line	Instruction
T0253	x^2
T0254	479
T0255	×
T0256	—
T0257	RCL T
T0258	4
T0259	y^x
T0260	179
T0261	×
T0262	+
T0263	RCL T
T0264	6
T0265	y^x
T0266	—
T0267	RCL× N
T0268	RCL F
T0269	COS
T0270	RCL× W
T0271	7
T0272	y^x
T0273	×
T0274	5040
T0275	÷
T0276	STO+ X
T0277	RCL K
T0278	STO× X
T0279	RCL J
T0280	STO+ X
****	Compute N
T0281	RCL F
T0282	COS
T0283	RCL× W
T0284	RCL× W
T0285	2
T0286	÷
T0287	STO Y
T0288	RCL P
T0289	x^2
T0290	4
T0291	×
T0292	RCL+ P
T0293	RCL T
T0294	x^2
T0295	—
T0296	RCL F

Line	Instruction
T0297	COS
T0298	x^3 †
T0299	×
T0300	RCL W
T0301	4
T0302	y^x
T0303	×
T0304	24
T0305	÷
T0306	STO+ Y
T0307	11
T0308	RCL T
T0309	x^2
T0310	24
T0311	×
T0312	—
T0313	8
T0314	×
T0315	RCL P
T0316	4
T0317	y^x
T0318	×
T0319	1
T0320	RCL T
T0321	x^2
T0322	6
T0323	×
T0324	—
T0325	28
T0326	×
T0327	RCL P
T0328	x^3 †
T0329	×
T0330	—
T0331	1
T0332	RCL T
T0333	x^2
T0334	32
T0335	×
T0336	—
T0337	RCL× P
T0338	RCL× P
T0339	+
T0340	RCL T
T0341	x^2

Line	Instruction
T0342	RCL× P
T0343	2
T0344	×
T0345	—
T0346	RCL T
T0347	4
T0348	y^x
T0349	+
T0350	RCL F
T0351	COS
T0352	5
T0353	y^x
T0354	×
T0355	RCL W
T0356	6
T0357	y^x
T0358	×
T0359	720
T0360	÷
T0361	STO+ Y
T0362	1385
T0363	RCL T
T0364	x^2
T0365	3111
T0366	×
T0367	—
T0368	RCL T
T0369	4
T0370	y^x
T0371	543
T0372	×
T0373	+
T0374	RCL T
T0375	6
T0376	y^x
T0377	—
T0378	RCL F
T0379	COS
T0380	7
T0381	y^x
T0382	×
T0383	RCL W
T0384	8
T0385	y^x
T0386	×

Line	Instruction
T0387	40320
T0388	÷
T0389	STO+ Y
T0390	RCL F
T0391	SIN
T0392	RCL× N
T0393	STO× Y
T0394	RCL M
T0395	STO+ Y
T0396	RCL K
T0397	STO× Y
T0398	RCL H
T0399	RCL× K
T0400	STO— Y
T0401	RCL I
T0402	STO+ Y
****	Compute γ
T0403	0
T0404	STO G
T0405	RCL F
T0406	SIN
T0407	RCL× W
T0408	STO— G
T0409	RCL P
T0410	x^2
T0411	2
T0412	×
T0413	RCL— P
T0414	RCL F
T0415	COS
T0416	x^2
T0417	×
T0418	RCL W
T0419	x^3 †
T0420	×
T0421	3
T0422	÷
T0423	RCL F
T0424	SIN
T0425	×
T0426	STO— G
T0427	11
T0428	RCL T
T0429	x^2
T0430	24

Line	Instruction
T0431	×
T0432	—
T0433	RCL P
T0434	4
T0435	y^x
T0436	×
T0437	11
T0438	RCL T
T0439	x^2
T0440	36
T0441	×
T0442	—
T0443	RCL P
T0444	x^3 †
T0445	×
T0446	—
T0447	1
T0448	RCL T
T0449	x^2
T0450	7
T0451	×
T0452	—
T0453	2
T0454	×
T0455	RCL× P
T0456	RCL× P
T0457	+
T0458	RCL T
T0459	x^2
T0460	RCL× P
T0461	+
T0462	RCL F
T0463	COS
T0464	4
T0465	y^x
T0466	×
T0467	RCL W
T0468	5
T0469	y^x
T0470	×
T0471	15
T0472	÷
T0473	RCL F
T0474	SIN
T0475	×

Line	Instruction
T0476	STO— G
T0477	17
T0478	RCL T
T0479	x^2
T0480	26
T0481	×
T0482	—
T0483	RCL T
T0484	4
T0485	y^x
T0486	2
T0487	×
T0488	+
T0489	RCL F
T0490	COS
T0491	6
T0492	y^x
T0493	×
T0494	RCL W
T0495	7
T0496	y^x
T0497	×
T0498	315
T0499	÷
T0500	RCL F
T0501	SIN
T0502	×
T0503	STO— G
T0504	RCL G
T0505	→DEG
T0506	→HMS
T0507	STO G
****	Compute k
T0508	1
T0509	STO S
T0510	RCL F
T0511	COS
T0512	RCL× W
T0513	x^2
T0514	2
T0515	÷
T0516	RCL× P
T0517	STO+ S
T0518	1
T0519	RCL T

Line	Instruction
T0520	x^2
T0521	6
T0522	×
T0523	—
T0524	RCL P
T0525	x^3 †
T0526	×
T0527	4
T0528	×
T0529	1
T0530	RCL T
T0531	x^2
T0532	24
T0533	×
T0534	+
T0535	RCL× P
T0536	RCL× P
T0537	+
T0538	RCL T
T0539	x^2
T0540	RCL× P
T0541	4
T0542	×
T0543	—
T0544	RCL F
T0545	COS
T0546	RCL× W
T0547	4
T0548	y^x
T0549	×
T0550	24
T0551	÷
T0552	STO+ S
T0553	61
T0554	RCL T
T0555	x^2
T0556	148
T0557	×
T0558	—
T0559	RCL T
T0560	4
T0561	y^x
T0562	16
T0563	×
T0564	+

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Line	Instruction
T0565	RCL F
T0566	COS
T0567	RCL× W
T0568	6
T0569	y^x
T0570	×
T0571	720
T0572	÷
T0573	STO+ S
T0574	RCL K
T0575	STO× S

Line	Instruction
T0576	RESULTS
T0577	PSE
T0578	EASTING
T0579	PSE
T0580	VIEW X
T0581	NORTHING
T0582	PSE
T0583	VIEW Y
T0584	GRID CONV
T0585	PSE
T0586	VIEW G

Line	Instruction
T0587	PT SCALE FACT
T0588	PSE
T0589	VIEW S
T0590	PROGRAM END
T0591	PSE
T0592	CF 10
T0593	RTN

Notes

(1) The program should be run in RPN mode, as results in ALG mode are unknown.

(2) Latitudes and longitudes should be entered in HP notation, i.e., DDD.MMSS. The grid convergence is displayed in HP notation.

(3) The program may be used for any Transverse Mercator projection, if the appropriate parameters are known. Similarly, any ellipsoid may be used, if its a and e^2 parameters are known. Parameters for a wide range of ellipsoids, all UTM zones and all SPCS TM zones are included at the end of this document.

(4) Latitudes in the southern hemisphere are negative. Longitudes west of Greenwich are negative, i.e., all longitudes in North America. It is critical to enter the correct sign in calculator when entering values.

(5) Lines with **** are comments only, and should not be entered into the calculator. They are there to make program entry a little easier.

(6) Because of the small type, there is the potential for confusing the 'square' x^2 and 'cube' x^3 instructions. Therefore 'cube' x^3 instruction lines include a † to make identification easier, i.e., x^3 †

(7) This program is long and often appears to be a stream of meaningless commands. This means that it may be more prone to errors when being entered. It is suggested that the program be entered using the given constants, tested (and the checksum checked), and when it is satisfactory, the constants at the start of the program can be changed to those most suitable for the bulk of the expected work.

(8) The program appears to work correctly, as tested. However, the grid convergence result has the opposite sign to that produced by the NGS on-line Lat/Long to SPCS conversion package at: http://www.ngs.noaa.gov/cgi-bin/spc_getpc.prl The formulae are correct in this program, and the results agree with the sign convention of Redfearn's formulae, as well as the normal usage of the grid convergence (converting between grid and true azimuths). I am not sure why the NGS program has the opposite sign, but I have asked NGS about it. Until this difference is resolved, be aware that the sign could be the opposite, and work out the correct sign from first principles.

Latitude/Longitude to Transverse Mercator Co-ordinates

Theory

Converting from geographical co-ordinates (latitude and longitude) to cartesian co-ordinates on a Transverse Mercator projection is a straightforward transformation, if somewhat long-winded.

Given that we have a, e^2, ϕ, k_0, λ and λ_0, we can use the following expressions for the conversion. These are Redfearn's Formulae. Note that these use an extra term in the computations of E' and N', compared to Snyder's book (1987), but this will make only a small difference in the overall values. The results will be a little different to the tabulated values for SPCS, too, owing to the limitations on the SPCS 27 computations. Remembering that the allowable distortion in the SPCS was to be no more than 1 in 10,000, it is acceptable to drop the final term in the formulae, as this doesn't degrade the formulae by anywhere near 1 in 10,000. These formulae will then agree with Snyder's formulae.

For UTM computations, you should use the full number of terms. This is because there is no 'legal' tolerance of distortion in the conversion process. UTM co-ordinates are now printed on 1:24,000 quadrangle maps, with either a grid/graticule or marginal ticks. These UTM co-ordinates are often on the NAD27 datum and need to be converted to NAD83 before they can be used. While there is a marginal note concerning the conversion of latitude and longitude from NAD27 to NAD83 on many of the more recent mapsheets, this value **does not** apply to the UTM co-ordinates (or the SPCS co-ordinates). This is because the latitude and longitude values are, in effect, figured from the origin in Kansas, while the UTM Northing co-ordinates are figured from the Equator. SPCS northings are figured from the zone origin, so will have a different shift for each zone. You should convert the co-ordinates to latitude and longitude for the appropriate system, convert these to NAD83, then convert to UTM or SPCS TM co-ordinates. An approximate set of shifts for UTM can be found in a paper by Welch, R., and Homsey, A., "Datum Shifts for UTM Co-ordinates," in the *Photogrammetric Engineering and Remote Sensing* journal, Volume 63, No. 4, pp. 371–375, published in 1997.

Conversion Formulae

Easting

$$\begin{aligned} E' = k_0 \{ & \nu \omega \cos\phi \\ & + \nu \frac{\omega^3}{6} \cos^3\phi \, (\psi - t^2) \\ & + \nu \frac{\omega^5}{120} \cos^5\phi \, [4\psi^3 (1 - 6t^2) + \psi^2 (1 + 8t^2) - \psi (2t^2) + t^4] \\ & + \nu \frac{\omega^7}{5040} \cos^7\phi \, (61 - 479t^2 + 179t^4 - t^6) \ \} \end{aligned}$$

Northing

$$\begin{aligned} N' = k_0 \{ m & + \nu \sin\phi \frac{\omega^2}{2} \cos\phi \\ & + \nu \sin\phi \frac{\omega^4}{24} \cos^3\phi \, (4\psi^2 + \psi - t^2) \\ & + \nu \sin\phi \frac{\omega^6}{720} \cos^5\phi \, [8\psi^4(11-24t^2) - 28\psi^3(1-6t^2) + \psi^2(1-32t^2) - 2\psi t^2 + t^4] \\ & + \nu \sin\phi \frac{\omega^8}{40320} \cos^7\phi \, (1385 - 3111t^2 + 543t^4 - t^6) \ \} \end{aligned}$$

Latitude/Longitude to Transverse Mercator Co-ordinates

Grid Convergence (in radians)

$$\gamma = -\sin\phi\ \omega - \sin\phi\ \frac{\omega^3}{3}\cos^2\phi\,(2\psi^2-\psi) - \sin\phi\ \frac{\omega^5}{15}\cos^4\phi\,[\psi^4(11-24t^2)-\psi^3(11-36t^2)+2\psi^2(1-7t^2)+\psi t^2] - \sin\phi\ \frac{\omega^7}{315}\cos^6\phi\,(17-26t^2+2t^4)$$

Point Scale Factor

$$k = k_0\left\{1 + \frac{\omega^2}{2}\cos^2\phi\ \psi + \frac{\omega^4}{24}\cos^4\phi\ [4\psi^3(1-6t^2)+\psi^2(1+24t^2)-4\psi t^2] + \frac{\omega^6}{720}\cos^6\phi\ (61-148t^2+16t^4)\right\}$$

where

$E' = E - E_0$ (E_0 is the offset of the central meridian; check the value for each zone. For UTM, E_0 = 500 000·000 meters.)

$N' = N - N_0$ (N_0 is the offset of the origin latitude; check the value for each zone. For UTM in the northern hemisphere, $N_0 = 0$; for UTM in the southern hemisphere, N_0 = 10 000 000·000 meters.)

ν = radius of curvature in the prime vertical at ϕ; i.e. $\nu = \dfrac{a}{\sqrt{1-e^2\sin^2\phi}}$

$\rho = \dfrac{a(1-e^2)}{(1-e^2\sin^2\phi)^{\frac{3}{2}}}$ = radius of curvature in the meridian at ϕ

$\omega = \lambda - \lambda_0$

$\psi = \dfrac{\nu}{\rho}$ i.e. ratio of the radii of curvature at ϕ

$t = \tan\phi$

m = meridian distance from equator, computed using the following expression

$m = a(A_0\phi - A_2\sin 2\phi + A_4\sin 4\phi - A_6\sin 6\phi)$

where ϕ is in radians and

$$A_0 = 1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256}$$

$$A_2 = \frac{3}{8}\left(e^2 + \frac{e^4}{4} + \frac{15e^6}{128}\right)$$

$$A_4 = \frac{15}{256}\left(e^4 + \frac{3e^6}{4}\right)$$

$$A_6 = \frac{35e^6}{3072}$$

Latitude/Longitude to Transverse Mercator Co-ordinates

With the appropriate values for ellipsoids and scale factors, these formulae will work for any Transverse Mercator projection: UTM, SPCS, or whatever.

a = semi-major axis of the ellipsoid; a = 6,378,137 m for WGS84 (GRS80)

e^2 = eccentricity of the ellipsoid; e^2 = 0.006 694 3800 for WGS84.

Sample Computations

Example 1

Using the SPCS 1983 (a = 6,378,137 m, e^2 = 0.006 694 3800), the following results are obtained.

Nevada East Zone, 2701, λ_0 = –115° 35', ϕ_0 = 34° 45', k_0 = 0.999 900, E_0 = 200,000.000 m, N_0 = 8,000,000.000 m.

Latitude = 41° 25' 00" Longitude = –115° 45' 20"

Easting (E) = 185,603.123 m Northing (N) = 8,739,929.417 m

Grid Convergence (γ) = 0° 06' 50.1" Point Scale Factor (k) = 0.999 902 55

Example 2

Using the SPCS 1927 (a = 20925832.2 ft, e^2 = 0.006 768 66), the following results are obtained.

Nevada East Zone, SPCS 1927, λ_0 = –115° 35', ϕ_0 = 34° 45', k_0 = 0.999 900, E_0 = 500,000.000 ft, N_0 = 0.000 ft.

Latitude = 41° 25' 00" Longitude = –115° 45' 20"

Easting (E) = 452,764.960 ft Northing (N) = 2,427,533.222 ft

Grid Convergence (γ) = 0° 06' 50.1" Point Scale Factor (k) = 0.999 902 55

Running the Program

Press XEQ T to start the program. The calculator briefly displays LAT—LONG 2 TM, then briefly shows CHECK—ENTER A. The program then stops and displays the prompt for entering the semi-major axis value, while displaying the current default value:

A?
6,378,137.0000 (This is for GRS80/WGS84/NAD83)

If you are happy with this value for the semi-major axis of the ellipsoid, press R/S to continue. Otherwise. Key in a different value (for a different ellipsoid) and press R/S to continue.

The calculator briefly displays CHECK—ENTER E. The program then stops and displays the prompt for entering the eccentricity of the ellipsoid, e:

E?
0.00669438 (This is for GRS80/WGS84/NAD83)

If this value for the eccentricity is correct, press R/S to continue. Otherwise, key in a different value (for a different ellipsoid) and press R/S to continue.

The calculator briefly displays CHECK—ENTER K. The program then stops and displays the prompt for entering the scale factor at the central meridian (λ_0), which is k_0:

K?
0.9996000 (This is for UTM)

If this value for the scale factor is satisfactory, press R/S to continue. If you want to change it, such as for a SPCS zone, key in the correct value and press R/S. In this case, key in 0.9999 for Nevada East (2701).

The calculator briefly displays CHK—NTR LONG 0. The program then stops and displays the prompt for entering the longitude of the central meridian of the projection, λ_0. Note that in the western hemisphere, this will be a negative value, and should be in HP notation (DDD.MMSS).

B?
—81.000000 (This is for UTM Zone 17)

If this is the correct central meridian, press R/S to continue, if this is not correct, key in the correct value, in HP notation, then press R/S to continue. In this case, key in –115.35 for Nevada East (2701).

The calculator briefly displays CHK—NTR LAT 0. The program then stops and displays the prompt for entering the latitude of the Northing co-ordinate origin, ϕ_0. For UTM, this is 0.000 (the equator), while for SPCS Zones, it is usually a latitude well south of the zone. The value should be entered in HP notation.

C?
0.000000 (This is for UTM)

If this is the correct latitude base, press R/S to continue. If you want a different value, key in that value and press R/S to continue. In this case, key in 34.45 and press R/S to continue.

The calculator briefly displays CHK—NTR N0. The program then stops and prompts for entry of the false northing value, or the northing offset. This is the value of the northing co-ordinate at ϕ_0, λ_0. For UTM in the northern hemisphere, this is 0.0000, while its value varies for different SPCS zones.

I?
0.0000 (This is for UTM)

If this is the correct value, press R/S to continue. If a different value is desired, key in the value and press R/S. In this case, key in 8000000.0 and press R/S. This is the N_0 value for Nevada East 2701.

The calculator briefly displays CHK—NTR E0. The program then stops and prompts for the false easting value, or the easting offset. This is the value of the easting at the central meridian (λ_0), denoted E_0.

J?
500,000.0000 (This is for UTM)

If this is the correct value, press R/S to continue. If a different value is required, key in the value and press R/S. In this case, key in 200000.000 and press R/S. This is the E_0 value for Nevada East 2701.

The calculator briefly displays ENTER LAT. The program stops and displays the prompt for entering the latitude of the point to be converted. This should be in HP notation.

F?
0.0000

Key in the latitude of the point in HP notation and press R/S to continue. In this case, key in 41.25 and press R/S.

The calculator briefly displays ENTER LONG. The program then stops and displays the prompt for entering the longitude of the point to be converted. This should be in HP notation.

L?
0.0000

Key in the longitude of the point in HP notation and press R/S to continue. In this case, key in —115.452 and press R/S.

The program displays RUNNING for a short while, then displays RESULTS briefly, followed by EASTING briefly. The program then stops and displays the easting value of the point. In this case, the calculator displays:

X=
185,603.1225

This is the easting of the point, in this case in meters. Press R/S to continue. The calculator briefly displays NORTHING, then stops and displays the northing value of the point. In this case, the calculator displays:

Y=
8,739,929.4173

This is the northing of the point, in this case in meters. Press R/S to continue. The calculator briefly displays GRID CONV, then stops and displays the grid convergence value in HP notation. In this case, the calculator displays:

G=
0.0650149

This is the grid convergence in HP notation, and is 0° 06' 50".149 in more conventional notation. Press R/S to continue. The calculator briefly displays PT SCALE FACT, then stops and displays the point scale factor of the point on the Transverse Mercator projection. In this case, the calculator displays:

S=
0.999902550

This is the point scale factor. Press R/S to continue. The calculator briefly displays PROGRAM END and then comes to an end, returning to the point from which it was called, or to normal operations.

Latitude/Longitude to Transverse Mercator Co-ordinates

Storage Registers Used

A	Semi-major axis of the ellipsoid being used, a
B	λ_0, the central meridian of the projection
C	ϕ_0, the origin latitude for the co-ordinates
D	temporary storage
E	Eccentricity of the ellipsoid, e^2
F	λ, latitude of the point to be converted
G	γ, the grid convergence of the point being converted
H	meridian distance of the origin latitude, ϕ_0
I	N_0, the offset for the northings (the northing at ϕ_0, λ_0)
J	E_0, the offset for the eastings (the easting at the central meridian, λ_0)
K	k_0, the scale factor at the central meridian, λ_0
L	λ, longitude of the point to be converted
M	m, meridian distance of the point to be converted
N	ν
P	$\psi = \frac{\nu}{\rho}$
R	ρ
S	k, point scale factor at the point being converted
T	$\tan \phi$
W	$\omega = \lambda - \lambda_0$
X	Easting co-ordinate of converted point
Y	Northing co-ordinate of converted point

Statistical Registers: not used

Labels Used

Label **T** Length = 3248 Checksum = 92BB

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

Flags Used

Flag 10 is set at the start of the program, and cleared at the end. This allows equations to be used as prompts and messages within the program.

Parameters for the Computations

Universal Transverse Mercator (UTM)

For UTM, the ϕ_0 value is 0° (the equator) for both northern and southern hemispheres. The λ_0 values are given for each zone in the table below.

Zone	Central Meridian, λ_0	Zone	Central Meridian, λ_0
1	177° W	31	3°E
2	171° W	32	9°E
3	165° W	33	15°E
4	159° W	34	21°E
5	153° W	35	27°E
6	147° W	36	33°E
7	141° W	37	39°E
8	135° W	38	45°E
9	129° W	39	51°E
10	123° W	40	57°E
11	117° W	41	63°E
12	111° W	42	69°E
13	105° W	43	75°E
14	99° W	44	81°E
15	93° W	45	87°E
16	87° W	46	93°E
17	81° W	47	99°E
18	75° W	48	105°E
19	69° W	49	111°E
20	63° W	50	117°E
21	57° W	51	123°E
22	51° W	52	129°E
23	45° W	53	135°E
24	39° W	54	141°E
25	33° W	55	147°E
26	27° W	56	153°E
27	21° W	57	159°E
28	15° W	58	165°E
29	9° W	59	171°E
30	3° W	60	177°E

The E_0 value for all zones is 500,000.000 meters. The N_0 value for the northern hemisphere is 0.000 meters. The N_0 value for the southern hemisphere is 10,000,000.000 meters.

State Plane Co-ordinate System (SPCS) 1983

Several US states use the Transverse Mercator projection for SPCS 1983. The various parameters for each zone in the 1983 system are given in the table below. Use these parameters with the program, together with the GRS80/WGS84/NAD83 ellipsoid parameters, in meters.

	Central Meridian	Latitude Origin	Central Scale	False Easting	False Northing
	λ_o	ϕ_0	k_0	E_0 (m)	N_0 (m)
Alabama					
East	85° 50'	30° 30'	0.9999600	200000.00	0.00
West	87° 30'	30° 00'	0.9999333	600000.00	0.00
Alaska					
2	142° 00'	54° 00'	0.9999000	500000.00	0.00
3	146° 00'	54° 00'	0.9999000	500000.00	0.00
4	150° 00'	54° 00'	0.9999000	500000.00	0.00
5	154° 00'	54° 00'	0.9999000	500000.00	0.00
6	185° 00'	54° 00'	0.9999000	500000.00	0.00
7	162° 00'	54° 00'	0.9999000	500000.00	0.00
8	166° 00'	54° 00'	0.9999000	500000.00	0.00
9	170° 00'	54° 00'	0.9999000	500000.00	0.00
Arizona					
East	110° 10'	31° 00'	0.9999000	213360.00	0.00
Central	111° 55'	31° 00'	0.9999000	213360.00	0.00
West	113° 45'	31° 00'	0.9999333	213360.00	0.00
Delaware	72° 25'	38° 00'	0.9999950	200000.00	0.00
Florida					
East	81° 00'	24° 20'	0.9999412	200000.00	0.00
West	82° 00'	24° 20'	0.9999412	200000.00	0.00
Georgia					
East	82° 10'	30° 00'	0.9999000	200000.00	0.00
West	84° 10'	30° 00'	0.9999000	700000.00	0.00
Hawaii					
1	155° 30'	18° 50'	0.9999667	500000.00	0.00
2	156° 40'	20° 20'	0.9999667	500000.00	0.00
3	158° 00'	21° 10'	0.9999900	500000.00	0.00
4	159° 30'	21° 50'	0.9999900	500000.00	0.00
5	160° 10'	21° 40'	1.0000000	500000.00	0.00

Latitude/Longitude to Transverse Mercator Co-ordinates

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (m)	False Northing N_0 (m)
Idaho					
East	112° 10'	41° 40'	0.9999474	200000.00	0.00
Central	114° 00'	41° 40'	0.9999474	500000.00	0.00
Illinois					
East	88° 20'	36° 40'	0.9999750	300000.00	0.00
West	90° 10'	36° 40'	0.9999412	700000.00	0.00
Indiana					
East	85° 40'	37° 30'	0.9999667	100000.00	250000.00
West	87° 05'	37° 30'	0.9999667	900000.00	250000.00
Maine					
East	68° 30'	43° 40'	0.9999000	300000.00	0.00
West	70° 10'	42° 50'	0.9999667	900000.00	0.00
Mississippi					
East	88° 50'	29° 30'	0.9999500	300000.00	0.00
West	90° 20'	29° 30'	0.9999500	700000.00	0.00
Missouri					
East	90° 30'	35° 50'	0.9999333	250000.00	0.00
Central	92° 30'	35° 50'	0.9999333	500000.00	0.00
West	94° 30'	36° 10'	0.9999412	850000.00	0.00
Nevada					
East	115° 35'	34° 45'	0.9999000	200000.00	8000000.00
Central	116° 40'	34° 45'	0.9999000	500000.00	6000000.00
West	118° 35'	34° 45'	0.9999000	800000.00	4000000.00
New Hampshire	71° 40'	42° 30'	0.9999667	300000.00	0.00
New Jersey	74° 30'	38° 50'	0.9999000	150000.00	0.00
New Mexico					
East	104° 20'	31° 00'	0.9999091	165000.00	0.00
Central	106° 15'	31° 00'	0.9999000	500000.00	0.00
West	107° 50'	31° 00'	0.9999167	830000.00	0.00

Latitude/Longitude to Transverse Mercator Co-ordinates

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (m)	False Northing N_0 (m)
New York					
East	74° 30'	40° 00'	0.9999000	150000.00	0.00
Central	76° 35'	40° 00'	0.9999375	250000.00	0.00
West	78° 35'	40° 00'	0.9999375	350000.00	0.00
Rhode Island	71° 30'	41° 05'	0.9999938	100000.00	0.00
Vermont	72° 30'	42° 30'	0.9999643	500000.00	0.00
Wyoming					
East	105° 10'	40° 30'	0.9999375	200000.00	0.00
East Central	107° 20'	40° 30'	0.9999375	400000.00	100000.00
West Central	108° 45'	40° 30'	0.9999375	600000.00	0.00
West	110° 05'	40° 30'	0.9999375	800000.00	100000.00

State Plane Co-ordinate System (SPCS) 1927

Several US states used the Transverse Mercator projection for SPCS 1927. The various parameters for each zone in the 1927 system are given in the table below. Use these parameters with the program, together with the Clarke 1866 ellipsoid in feet.

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Alabama					
East	85° 50'	30° 30'	0.9999600	500000.00	0.00
West	87° 30'	30° 00'	0.9999333	500000.00	0.00
Alaska					
2	142° 00'	54° 00'	0.9999000	500000.00	0.00
3	146° 00'	54° 00'	0.9999000	500000.00	0.00
4	150° 00'	54° 00'	0.9999000	500000.00	0.00
5	154° 00'	54° 00'	0.9999000	500000.00	0.00
6	185° 00'	54° 00'	0.9999000	500000.00	0.00
7	162° 00'	54° 00'	0.9999000	700000.00	0.00
8	166° 00'	54° 00'	0.9999000	500000.00	0.00
9	170° 00'	54° 00'	0.9999000	600000.00	0.00

Latitude/Longitude to Transverse Mercator Co-ordinates

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Arizona					
East	110° 10'	31° 00'	0.9999000	500000.00	0.00
Central	111° 55'	31° 00'	0.9999000	500000.00	0.00
West	113° 45'	31° 00'	0.9999333	500000.00	0.00
Delaware	72° 25'	38° 00'	0.9999950	500000.00	0.00
Florida					
East	81° 00'	24° 20'	0.9999412	500000.00	0.00
West	82° 00'	24° 20'	0.9999412	500000.00	0.00
Georgia					
East	82° 10'	30° 00'	0.9999000	500000.00	0.00
West	84° 10'	30° 00'	0.9999000	500000.00	0.00
Hawaii					
1	155° 30'	18° 50'	0.9999667	500000.00	0.00
2	156° 40'	20° 20'	0.9999667	500000.00	0.00
3	158° 00'	21° 10'	0.9999900	500000.00	0.00
4	159° 30'	21° 50'	0.9999900	500000.00	0.00
5	160° 10'	21° 40'	1.0000000	500000.00	0.00
Idaho					
East	112° 10'	41° 40'	0.9999474	500000.00	0.00
Central	114° 00'	41° 40'	0.9999474	500000.00	0.00
West	115° 45'	41° 40'	0.9999333	500000.00	0.00
Illinois					
East	88° 20'	36° 40'	0.9999750	500000.00	0.00
West	90° 10'	36° 40'	0.9999412	500000.00	0.00
Indiana					
East	85° 40'	37° 30'	0.9999667	500000.00	0.00
West	87° 05'	37° 30'	0.9999667	500000.00	0.00
Maine					
East	68° 30'	43° 50'	0.9999000	500000.00	0.00
West	70° 10'	42° 50'	0.9999667	500000.00	0.00

Latitude/Longitude to Transverse Mercator Co-ordinates

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Michigan (old)					
East	83° 40'	41° 30'	0.9999429	500000.00	0.00
Central	85° 45'	41° 30'	0.9999091	500000.00	0.00
West	88° 45'	41° 30'	0.9999091	500000.00	0.00
Mississippi					
East	88° 50'	29° 40'	0.9999600	500000.00	0.00
West	90° 20'	30° 30'	0.9999412	500000.00	0.00
Missouri					
East	90° 30'	35° 50'	0.9999333	500000.00	0.00
Central	92° 30'	35° 50'	0.9999333	500000.00	0.00
West	94° 30'	36° 10'	0.9999412	500000.00	0.00
Nevada					
East	115° 35'	34° 45'	0.9999000	500000.00	0.00
Central	116° 40'	34° 45'	0.9999000	500000.00	0.00
West	118° 35'	34° 45'	0.9999000	500000.00	0.00
New Hampshire	71° 40'	42° 30'	0.9999667	500000.00	0.00
New Jersey	74° 40'	38° 50'	0.9999750	2000000.00	0.00
New Mexico					
East	104° 20'	31° 00'	0.9999091	500000.00	0.00
Central	106° 15'	31° 00'	0.9999000	500000.00	0.00
West	107° 50'	31° 00'	0.9999167	500000.00	0.00
New York					
East	74° 20'	40° 00'	0.9999667	500000.00	0.00
Central	76° 35'	40° 00'	0.9999375	500000.00	0.00
West	78° 35'	40° 00'	0.9999375	500000.00	0.00
Rhode Island	71° 30'	41° 05'	0.9999938	500000.00	0.00
Vermont	72° 30'	42° 30'	0.9999643	500000.00	0.00

Latitude/Longitude to Transverse Mercator Co-ordinates

	Central Meridian λ_o	Latitude Origin ϕ_0	Central Scale k_0	False Easting E_0 (ft)	False Northing N_0 (ft)
Wyoming					
East	105° 10'	40° 40'	0.9999412	500000.00	0.00
East Central	107° 20'	40° 40'	0.9999412	500000.00	0.00
West Central	108° 45'	40° 40'	0.9999412	500000.00	0.00
West	110° 05'	40° 40'	0.9999412	500000.00	0.00

Ellipsoids

There are a range of ellipsoids in common or former use. The table below has the a and e^2 values for a number of common (and less common) ellipsoids.

Ellipsoid	a Semi-major Axis	e^2 Eccentricity
GRS80–WGS94–NAD83	6378137 m	0.006 694 38
Clarke 1866 (NAD27)	6378206.4 m	0.006 768 66
Clarke 1866 (NAD27)	20925832.2 ft	0.006 768 66
ANS (Australian)	6378160 m	0.006 694 541 855
Airy 1830	6377563.4 m	0.006 670 54
Bessel 1841	6377397.16 m	0.006 674 372
Clarke 1880	6378249.15 m	0.006 803 511
Everest 1830	6377276.35 m	0.006 637 847
Fischer 1960 (Mercury)	6378166 m	0.006 693 422
Fischer 1968	6378150 m	0.006 693 422
Hough 1956	6378270 m	0.006 722 67
International	6378388 m	0.006 722 67
Krassovsky 1940	6378245 m	0.006 693 422
South American 1960	6378160 m	0.006 694 542
GRS 1967	6378160 m	0.006 694 605
GRS 1975	6378140 m	0.006 694 385
WGS 60	6378165 m	0.006 693 422
WGS 66	6378145 m	0.006 694 542
WGS 72	6378135 m	0.006 694 317 778
WGS 84	6378137 m	0.006 694 38

Reference

SNYDER, J.P., 1987. *Map Projections—A Working Manual*. U.S. Geological Survey Professional Paper 1395. Washington: US Government Printing Office.