

**Transverse Mercator Traverse Computations**

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Line	Instruction	Line	Instruction	Line	Instruction
T0001	LBL T	U0022	STOP	U0061	x <> y
T0002	CL $\Sigma$	U0023	RCL U	U0062	x < 0 ?
T0003	INPUT K	U0024	RCL- E	U0063	RCL+ Z
T0004	INPUT E	U0025	3	U0064	→HMS
T0005	INPUT N	U0026	×	U0065	x <> y
T0006	INPUT L	U0027	RCL+ I	U0066	STOP
T0007	INPUT B	U0028	RCL+ I	U0067	RCL V
T0008	INPUT X	U0029	RCL× J	U0068	RCL- Y
T0009	INPUT Y	U0030	8.467E-10	U0069	RCL U
T0010	RCL X	U0031	RCL B	U0070	RCL- X
T0011	STO U	U0032	×	U0071	STOP
T0012	RCL Y	U0033	×	U0072	XEQ C
T0013	STO V	U0034	STOP	U0073	STO D
T0014	XEQ H	U0035	RCL I	U0074	STO I
T0015	STOP	U0036	STO+ U	U0075	STO J
T0016	XEQ S	U0037	RCL J	U0076	STO S
T0017	STO S	U0038	STO+ V	U0077	STO U
T0018	STOP	U0039	XEQ H	U0078	STO V
U0001	LBL U	U0040	STOP	U0079	STO W
U0002	→HR	U0041	XEQ S	U0080	RTN
U0003	STOP	U0042	STOP		
U0004	STO D	U0043	RCL S	S0001	LBL S
U0005	$\theta, r \rightarrow y, x$	U0044	x <> y	S0002	RCL U
U0006	STO J	U0045	STO S	S0003	RCL- E
U0007	x <> y	U0046	+	S0004	x <sup>2</sup>
U0008	STO I	U0047	2	S0005	1.23 E-14
U0009	x <> y	U0048	÷	S0006	RCL B
U0010	$\Sigma+$	U0049	STOP	S0007	÷
U0011	RCL U	U0050	RCL D	S0008	×
U0012	RCL- E	U0051	×	S0009	RCL+ K
U0013	3	U0052	STOP	S0010	RTN
U0014	×	U0053	RCL U		
U0015	RCL I	U0054	RCL V		
U0016	+	U0055	STOP		
U0017	RCL× J	U0056	x ≥ 0 ?		
U0018	-8.467 E-10	U0057	GTO U		
U0019	RCL B	U0058	$\Sigma y$		
U0020	×	U0059	$\Sigma x$		
U0021	×	U0060	y, x → $\theta, r$		

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Line	Instruction
H0001	LBL H
H0002	RCL V
H0003	RCL- N
H0004	111120
H0005	RCL B
H0006	$\sqrt{x}$
H0007	$\div$
H0008	$\div$
H0009	RCL L
H0010	$\rightarrow$ HR
H0011	+
H0012	STO W
H0013	RCL U
H0014	RCL- E
H0015	111120
H0016	RCL B
H0017	$\sqrt{x}$
H0018	$\div$
H0019	$\div$
H0020	RCL W
H0021	COS
H0022	$\div$
H0023	TAN
H0024	RCL W
H0025	SIN
H0026	$\times$
H0027	+/-
H0028	ATAN
H0029	$\rightarrow$ HMS
H0030	RTN

**Notes**

- (1) This is a program to deal with a traverse that is to be brought onto the SPCS. The idea is to enter all the required data to compute the corrections to bring the data onto the SPCS, and do a preliminary traverse closure at the same time.
- (2) The program consists of three blocks of code. The first is the main program, which contains sub-blocks T and U. These need to be entered sequentially, as control runs from one to the next. Sub-block T contains the initial data entry part, while sub-block U contains the main loop and computation part, as well as the finishing and clean-up part.

There are two subroutines (actually functions), which can be located anywhere in program memory. They require certain values to be in memory registers and are called by the main program to compute specific values. Block H computes the grid convergence at the given point. Block S computes the point scale factor at the given point.

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- (3) The stack is used very little for data return. In most cases, a single value is presented in the X-register at a time. The only exceptions are when the co-ordinates of a point are presented, or the final misclosure information is shown.
- (4) Labeling is problematic on the HP-33S. Outputs are not labeled. Some inputs are labeled, but this is mostly the initial inputs. During the main part of the program there is no labeling, so you will have to follow the instructions fairly carefully.
- (5) The program will work properly in either SPCS 1983 or SPCS 1927, with the proviso that things may be computed slightly differently from SPCS 1927, owing to differences in the computations that produced the tables. If using SPCS 1927, use feet for the inputs and enter 0.092903 into B. This corrects the arc-to-chord correction computation and scale factor to work in feet. Note that the tabulations of data supplied via the websites associated with this program have the 1927 data in meters.

Similarly you can work in feet (of whatever type) in SPCS 1983, provided you compute the correct value of the two parameters for the zone, i.e.,  $E_0$  and  $N_0$ . But it is best to use meters for SPCS 1983.

- (6) Bearings and angles are entered and displayed in HP notation, i.e., DDD.MMSS
- (7) The misclosure components in X (or E) and Y (or N) can be displayed by recalling  $\Sigma y$  and  $\Sigma x$  using the SUMS menu. (Note these are back-to-front.) Reversing their order using  $x \leftrightarrow y$  sets them up for conversion to polar. These can then be compared with the co-ordinate differences displayed as the misclosure components, which provide the output on misclosure components from the program.

**Theory**

The program works by running a small traverse closure routine and computing:

- the grid convergence ( $\gamma$ ) at each point;
- the point scale factor ( $k$ ) at each point;
- the co-ordinates of each point (E, N or X, Y);
- the line scale factor ( $K$ ) of each line;
- the grid length ( $L$ ) of each line; and
- the arc-to-chord corrections ( $\delta$ ) for each end of each line.

You enter all the data required to complete the above computations. Formulae are as per the SPCS Notes on the Transverse Mercator Projection. Latitude and longitude difference are computed by dividing the  $\Delta N$  and  $E'$  values for each point by 111120 to convert meters to degrees of latitude and correcting longitude differences for the latitude. Approximate latitudes are sufficient for computing the grid convergence. Meters are converted to feet internally for the various computations, according to the value in the B memory register.

It is necessary to convert the bearings of the traverse to grid bearings before using them in the program. This will ensure that the co-ordinates and associated data are calculated correctly. After entering the grid co-ordinates of the first point, the grid convergence is displayed, and this value can be used to convert bearings on a true azimuth to an approximate grid azimuth. If you run the program only to this point, you can then stop and compute all the grid bearings, then restart the program. As all that has been done is data entry, these values should still be in place and can be accepted by pressing R/S quite quickly.

Whole circle bearings in HP notation are used. The program uses no error checking on entered data. Line scale factor is computed by taking mean of the point scale factors of both of the line.

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**Sample Computation**

For this computation, we will use the data from the SPCS tables that are currently available on the <http://homepage.mac.com/nwjh/HP-33S/> website. These are in an Excel spreadsheet or PDF files. Make sure that you are using the correct projection.

Using the example below on Indiana East Zone 1983, we first compute all the grid bearings, which is straightforward as the given line already has a grid bearing.

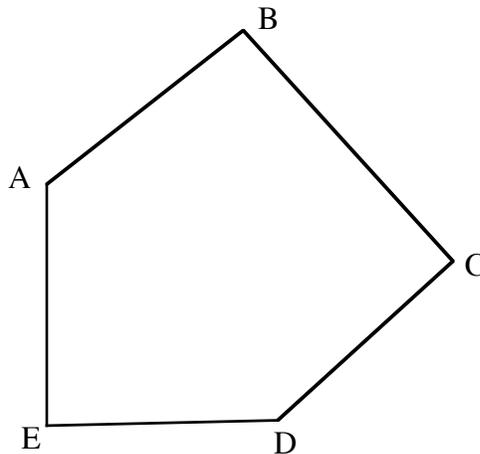
Transverse Mercator Projection, SPCS 1983, Indiana East Zone (from the tables):

Central scale factor,  $k_0$                     0.999 9667  
 Origin                                        37° 30' N                    [ 85° 40' W (central meridian,  $\lambda_0$ ) (not required)]  
 Co-ordinates of origin                     $E_0 = 600\,000\cdot000$                      $N_0 = 0\cdot000$  (in meters) (false origin)

Point A has the following co-ordinates.

E 97 763·227                    N 574 629·037

The grid bearing of the line from A to B is 47° 51' 27".



The interior angles at each traverse point, as observed in the field, are as follows:

A            132° 18' 25"  
 B            87° 23' 19"  
 C            85° 08' 08"  
 D            140° 54' 40"  
 E            94° 15' 30"

The lengths of the sides, reduced to ellipsoidal distances, are as follows:

AB            14 302·785  
 BC            12 821·076  
 CD            15 093·269  
 DE            6 394·974  
 EA            8 383·815

As we are working in meters, we will enter 1.0 for the value in the B register.

**Transverse Mercator Traverse Computations**

Begin by computing the grid bearings around the traverse. The bearings are:

AB	47° 51' 27"
BC	140° 28' 08"
CD	235° 20' 00"
DE	274° 25' 20"
EA	0° 09' 50"

There is a 2" angular misclosure in the traverse, but we will ignore this for this example, especially as 1" of that is spherical excess that should be taken care of by the arc-to-chord corrections.

**Start the program** by pressing XEQ T

*You first enter data about the particular zone you are in.*

Prompt:	K?	Enter $k_0$	0.9999667	Press R/S
Prompt:	E?	Enter $E_0$	100000.0	Press R/S
Prompt:	N?	Enter $N_0$	250000.0	Press R/S
Prompt:	L?	Enter $r_0$	37.30 (37° 30')	Press R/S
Prompt:	B?	Enter ft or m value	1 (for m)	Press R/S

*The next entries are about your starting point, A.*

Prompt:	X?	Enter E or X	97763.227	Press R/S
Prompt:	Y?	Enter N or Y	574629.037	Press R/S

*The program now presents the grid convergence at A.*

0.01017 (0° 01' 01.7") Press R/S

*The program now presents the point scale factor at A.*

0.9999668 Don't press R/S yet!

You can use the grid convergence for the starting point to compute grid bearings for the traverse, if necessary, but outside this program.

# HP-33S Calculator Program

## Transverse Mercator Traverse Computations

TM 1

*Enter the data for side 1, A to B.* (This is the start of the main program loop)

Enter the bearing of the side: 47.5127 (47° 51' 27") Now press R/S

Enter the length of the side: 14302.785 Press R/S

*The program now presents the arc-to-chord correction at A going to B*

$\delta_{AB} = -0.03$  (in seconds) Press R/S

*The program now presents the arc-to-chord correction at B going to A.*

$\delta_{BA} = +0.12$  (in seconds) Press R/S

*The program now presents the grid convergence at B.*

$\gamma = -0.03516$  ( $-0^\circ 03' 51.6''$ ) Press R/S

*The program now presents the point scale factor at B.*

$k = 0.999\ 9676$  Press R/S

*The program now presents the line scale factor for the line A to B.*

$K = 0.999\ 9672$  Press R/S

*The program now presents the grid distance for the line A to B.*

$L = 14302.315$  Press R/S

*The program now presents the co-ordinates of point B in the Y and X registers of the stack.*

E 108368.432 (in the Y register)

N 584225.874 (in the X register) Don't press R/S yet!

Note that this line crosses the central meridian and so may be subject to being S-shaped. In this case, this doesn't appear to have happened. However, the line is curved away from the central meridian, but the part to the east dominates, so the line curves away to the east for its entire length. This is a little counter-intuitive, but the corrections are small.

The program uses the simplified arc-to-chord correction formula from both ends of the line, obtaining  $\delta_{AB} = -0.03''$  and  $\delta_{BA} = +0.12''$ . These have the correct sign, but the change from end to end is noticeable. If we were to use the more rigorous formula, we may get slightly different results again.

On a Transverse Mercator projection, the S-shaped line problem only seems to occur when the line is split in the ratio of about 1:2 across the central meridian. Even then, for most lines, the arc-to-chord corrections are very small.

**HP-33S Calculator Program**  
**Transverse Mercator Traverse Computations**

**TM 1**

*Enter the data for side 2, B to C.*

Enter the bearing of the side: 140.2808 (140° 28' 08") Now press R/S

Enter the length of the side: 12821.076 Press R/S

*The program now presents the arc-to-chord correction at B going to C*

$\delta_{BC} = +0.28$  (in seconds) Press R/S

*The program now presents the arc-to-chord correction at C going to B.*

$\delta_{CB} = -0.35$  (in seconds) Press R/S

*The program now presents the grid convergence at C.*

$\gamma = -0.0736$  (-0° 07' 36") Press R/S

*The program now presents the point scale factor at C.*

$k = 0.999\ 9701$  Press R/S

*The program now presents the line scale factor for the line B to C.*

$K = 0.999\ 9688$  Press R/S

*The program now presents the grid distance for the line B to C.*

$L = 12820.676$  Press R/S

*The program now presents the co-ordinates of point C in the Y and X registers of the stack.*

E 116529.010 (in the Y register)

N 574337.246 (in the X register) Don't press R/S yet!

**HP-33S Calculator Program**  
**Transverse Mercator Traverse Computations**

**TM 1**

*Enter the data for side 3, C to D.*

Enter the bearing of the side: 235.2000 (235° 20' 00") Now press R/S

Enter the length of the side: 15093.269 Press R/S

*The program now presents the arc-to-chord correction at C going to D.*

$\delta_{CD} = +0.27$  (in seconds) Press R/S

*The program now presents the arc-to-chord correction at D going to C.*

$\delta_{DC} = -0.18$  (in seconds) Press R/S

*The program now presents the grid convergence at D.*

$\gamma = -0.0153$  (-0° 01' 53") Press R/S

*The program now presents the point scale factor at D.*

$k = 0.999\ 9669$  Press R/S

*The program now presents the line scale factor for the line C to D.*

$K = 0.999\ 9685$  Press R/S

*The program now presents the grid distance for the line C to D.*

$L = 15092.793$  Press R/S

*The program now presents the co-ordinates of point D in the Y and X registers of the stack.*

E 104115.172 (in the Y register)

N 565752.178 (in the X register)

Don't press R/S yet!

**HP-33S Calculator Program**  
**Transverse Mercator Traverse Computations**

**TM 1**

*Enter the data for side 4, D to E.*

Enter the bearing of the side: 274.2520 (274° 25' 20") Now press R/S

Enter the length of the side: 6394.974 Press R/S

*The program now presents the arc-to-chord correction at D going to E.*

$\delta_{DE} = -0.002$  (in seconds) Press R/S

*The program now presents the arc-to-chord correction at E going to D.*

$\delta_{ED} = -0.0002$  (in seconds) Press R/S

*The program now presents the grid convergence at E.*

$\gamma = 0.0102$  (0° 01' 02") Press R/S

*The program now presents the point scale factor at E.*

$k = 0.999\ 9668$  Press R/S

*The program now presents the line scale factor for the line D to E.*

$K = 0.999\ 9668$  Press R/S

*The program now presents the grid distance for the line D to E.*

$L = 6394.762$  Press R/S

*The program now presents the co-ordinates of point E in the Y and X registers of the stack.*

E 97739.237 (in the Y register)

N 566245.267 (in the X register) Don't press R/S yet!

Note that east-west lines have no arc-to-chord correction on a Transverse Mercator projection. This line crosses the central meridian, but there is almost no noticeable effect (i.e., almost no S-shaped distortion), as it is almost exactly east-west. There is a tiny effect (both corrections the same sign, albeit minute), but we may have to use a more rigorous arc-to-chord correction formula to be sure that it is real.

# HP-33S Calculator Program

## Transverse Mercator Traverse Computations

TM 1

*Enter the data for side 5, E to A.*

Enter the bearing of the side: 0.0950 (0° 09' 50") Now press R/S

Enter the length of the side: 8383.815 Press R/S

*The program now presents the arc-to-chord correction at E going to A.*

$\delta_{EA} = +0.05$  (in seconds) Press R/S

*The program now presents the arc-to-chord correction at A going to E.*

$\delta_{AE} = -0.05$  (in seconds) Press R/S

*The program now presents the grid convergence at A.*

$\gamma = 0.01017$  (0° 01' 01.7") Press R/S

*The program now presents the point scale factor at A.*

$k = 0.999\ 9668$  Press R/S

*The program now presents the line scale factor for the line E to A.*

$K = 0.999\ 9668$  Press R/S

*The program now presents the grid distance for the line E to A.*

$L = 8383.536$  Press R/S

*The program now presents the co-ordinates of point A in the Y and X registers of the stack.*

E 97763.218 (in the Y register)

N 574629.048 (in the X register) Don't press R/S yet!

### ***Finish and clean up.***

As we are now finished the traverse, enter a negative bearing to jump out of the main loop.

Enter -1 Now Press R/S

*The program presents the misclosure in the X and Y registers of the stack.*

In Y register 318.4729 (318° 47') Bearing of misclosure

In X register 0.014 Length of misclosure Press R/S

*The program now presents the misclosure components, computed by comparing co-ordinates.*

In Y register -0.009  $\Delta E$

In X register 0.011  $\Delta N$

At this point the program has completed all its required operations. If you press R/S once more, the program will clear the temporary variables from the memory registers, but leave the main data, in case you want to re-compute the traverse, or another in the same zone.

### **Final Computations**

You now have the arc-to-chord corrections for the whole traverse, together with the point and line scale factors and the grid distances. If the arc-to-chord corrections are large enough to justify recomputing the bearings, do this manually. Then run the data (starting co-ordinates, grid

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bearings and grid distances) through any closure program to compute the final co-ordinates. By converting all your data to plane bearings and distances, all computations can be undertaken with plane computational methods.

The solution of the arc-to-chord corrections, showing the shape of the measured geodetic lines, as projected onto the Transverse Mercator map projection, is shown on the diagram on the next page.

So, to compute the corrected grid bearings for the final adjustment, we get the following solution.

Grid Bearing B to A	227° 51' 27"	
	<u>-0.1"</u>	(chord-to-arc correction)
	227° 51' 26.9"	
Observed Angle @ B	<u>87° 23' 19"</u>	
	140° 28' 07.9"	
	<u>+0.3"</u>	(arc-to-chord correction)
Grid Bearing B to C	140° 28' 08.2"	
Grid Bearing C to B	320° 28' 08.2"	
	<u>-0.4"</u>	(chord-to-arc correction)
	320° 28' 07.8"	
Observed Angle @ C	<u>85° 08' 08"</u>	
	235° 19' 59.8"	
	<u>+0.3"</u>	(arc-to-chord correction)
Grid Bearing C to D	235° 20' 00.1"	
Grid Bearing D to C	55° 20' 00.1"	
	<u>+0.2"</u>	(chord-to-arc correction)
	55° 20' 00.3"	
Observed Angle @ D	<u>140° 54' 40"</u>	
	274° 25' 20.3"	
	<u>0.0"</u>	(arc-to-chord correction)
Grid Bearing D to E	274° 25' 20.3"	
Grid Bearing E to D	94° 25' 20.3"	
	<u>0.0"</u>	(chord-to-arc correction)
	94° 25' 20.3"	
Observed Angle @ E	<u>94° 15' 30"</u>	
	0° 09' 50.3"	
	<u>+0.1"</u>	(arc-to-chord correction)
Grid Bearing E to A	0° 09' 50.4"	
Grid Bearing A to E	180° 09' 50.4"	
	<u>+0.1"</u>	(chord-to-arc correction)
	180° 09' 50.5"	
Observed Angle @ A	<u>132° 18' 25"</u>	
	47° 51' 25.5"	
	<u>0.0"</u>	(arc-to-chord correction)
Grid Bearing A to B	47° 51' 25.5"	

Comparing the closing bearing to the initial bearing, we see that the difference is 1.5". Since we computed the spherical excess for this traverse to be 1.1", and the overall angular misclosure to be 2", we can see that the arc-to-chord corrections have eliminated the spherical excess from the traverse, although we appear to have left some rounding errors in there, about 0.6". This is hardly significant, but we could deal with it by carrying more figures in the arc-to-chord corrections.

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The above grid bearings would now be used to compute the final co-ordinates, using the starting co-ordinates of point A and the grid distances computed during the program's run. These values would be used in an ordinary traverse closure program, e.g., Closure 2 for co-ordinates or Closure 1 for area within the polygon. Everything from here is done using plane methods. The traverse may also be adjusted at this stage, using a suitable method, e.g., least squares.

**Storage Registers Used**

<b>B*</b>	1.0 or 0.0929 (feet conversion for 1927)	<b>N*</b>	$N_0$ false origin north
<b>D</b>	length of current line	<b>S*</b>	point scale factor at point
<b>E*</b>	$E_0$ false origin east	<b>U</b>	X or E of current point
<b>I</b>	DE of current line	<b>V</b>	Y or N of current point
<b>J</b>	DN of current line	<b>W</b>	approx. $\phi$ of current point
<b>K*</b>	$k_0$ central point scale factor	<b>X</b>	X or E of starting point
<b>L*</b>	$\phi_0$ latitude of origin	<b>Y</b>	Y or N of starting point

Statistical Registers:  $\Sigma x$  = Current  $\Delta Y$  or  $\Delta N$  from starting point

$\Sigma y$  = Current  $\Delta X$  or  $\Delta E$  from starting point

**Labels Used**

Label <b>H</b>	Length = 114	Checksum = 4897
Label <b>S</b>	Length = 42	Checksum = 44E4
Label <b>T</b>	Length = 54	Checksum = E9DA
Label <b>U</b>	Length = 300	Checksum = ACE7

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.