Three-Point Horizontal Resection Reduction Program

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Date: October, 2007. **Version**: 1.0 **Mnemonic**: R for Resection

Line	Instruction	Display	User Programming Instructions
R001	LBL R		
R002	CLSTK		
R003	FS? 10		
R004	GTO R008		
R005	SF 1		
R006	SF 10		
R007	GTO R009		
R008	CF 1		
R009	RESECTION		
R010	PSE		
R011	ENTER LEFT X		(Key in using EQN RCL E RCL N etc.)
R012	PSE		
R013	INPUT X		
R014	STO A		
R015	ENTER LEFT Y		(Key in using EQN RCL E RCL N etc.)
R016	PSE		
R017	INPUT Y		
R018	STO B		
R019	ENTER MID X		(Key in using EQN RCL E RCL N etc.)
R020	PSE		
R021	INPUT X		
R022	STO C		
R023	ENTER MID Y		(Key in using EQN RCL E RCL N etc.)
R024	PSE		
R025	INPUT Y		
R026	STO D		
R027	ENTER RIGHT X		(Key in using EQN RCL E RCL N etc.)
R028	PSE		
R029	INPUT X		
R030	STO E		
R031	ENTER RIGHT Y		(Key in using EQN RCL E RCL N etc.)
R032	PSE		
R033	INPUT Y		
R034	STO F		
R035	ENTER ALPHA		(Key in using EQN RCL E RCL N etc.)
R036	PSE		
R037	INPUT X		
R038	HMS→		
R039	STO G		

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R040	ENTER BETA		(Key in using EQN RCL E RCL N etc.)
R041	PSE		
R042	INPUT X		
R043	HMS→		
R044	STO H		
R045	RCL B		
R046	RCL– D		
R047	RCL A		
R048	RCL– C		
R049	0 **i** 1		Press the zero key, then **i**, then 1.
R050	×		
R051	+		
R052	STO L		
R053	RCL F		
R054	RCL– D		
R055	RCL E		
R056	RCL– C		
R057	0 **i** 1		Press the zero key, then **i**, then 1.
R058	×		
R059	+		
R060	STO K		
R061	360		
R062	STO Z		
R063	RCL L		
R064	ARG		
R065	RCL K		
R066	ARG		
R067	–		
R068	x < 0 ?		
R069	RCL+ Z		
R070	STO I		
R071	RCL+ G		
R072	RCL+ H		
R073	RCL Z		
R074	x < > y		
R075	–		
R076	STO S		
R077	RCL L		
R078	ABS		
R079	RCL H		
R080	SIN		
R081	×		
R082	RCL K		
R083	ABS		
R084	÷		
R085	RCL G		
R086	SIN		

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R087	÷		
R088	RCL S		
R089	SIN		
R090	÷		
R091	RCL S		
R092	TAN		
R093	1/x		
R094	+		
R095	1/x		
R096	ATAN		
R097	STO X		
R098	RCL L		
R099	ARG		
R100	180		
R101	+		
R102	RCL+ G		
R103	RCL+ X		
R104	STO Y		
R105	RCL L		
R106	ABS		
R107	RCL X		
R108	SIN		
R109	×		
R110	RCL G		
R111	SIN		
R112	÷		
R113	STO J		
R114	RCL Y		
R115	SIN		
R116	×		
R117	STO P		
R118	RCL J		
R119	RCL Y		
R120	COS		
R121	×		
R122	STO Q		
R123	RCL P		
R124	RCL+ C		
R125	STO X		
R126	UNKNOWN X =		(Key in using EQN RCL U RCL N etc.)
R127	PSE		
R128	VIEW X		
R129	UNKNOWN Y =		(Key in using EQN RCL U RCL N etc.)
R130	PSE		
R131	RCL Q		
R132	RCL+ D		
R133	STO Y		

R134	VIEW Y		
R135	RCL I		
R136	RCL+ G		
R137	RCL+ H		
R138	→HMS		
R139	CHECK VALUE =		(Key in using EQN RCL C RCL H etc.)
R140	PSE		
R141	STOP		
R142	FS? 1		
R143	CF 10		
R144	RTN		

Notes

(1) Horizontal 3-point resection solution, based on measuring two angles between three known points at an unknown point, the location of which is to be computed.

(2) Brief prompts are provided before each requirement for data entry, as well as before results are displayed. Each prompt shows for about 1 second, and is then replaced by the value or request for input.

(3) Co-ordinates of the unknown point are displayed following brief prompts. They are also stored in storage registers X and Y for later retrieval.

(4) Angles are entered and displayed in HP notation, i.e., DDD.MMSS. Internal storage of angles and azimuths is in decimal degrees. Internal storage of lines uses the calculator's complex number format.

Theory

This 2-D resection uses Ormsby's solution. In the discussion below, A is the left point, B is the middle point, C is the right point, and P is the unknown point. The left angle is alpha (α) and the right angle is beta (β). The interior angle at B is gamma (γ). The angle at point A is x, which is the first objective of the solution. A diagram is shown on the next page.

α and β are angles observed from the point P to points A, B and C, whose co-ordinate are known.

$$BP = \frac{AB \sin x}{\sin \alpha} = \frac{BC \sin y}{\sin \beta}$$

and $(x + y) = (360° - (\alpha + \beta + \gamma)) = s$

$$\frac{AB}{\sin \alpha} \sin x = \frac{BC}{\sin \beta} \sin (s - x) = \frac{BC}{\sin \beta}(\sin s \cos x - \cos s \sin x)$$

$$\frac{AB}{\sin \alpha} \sin x = \frac{BC}{\sin \beta} \sin s \cos x - \frac{BC}{\sin \beta} \cos s \sin x$$

$$\sin x \left(\frac{AB}{\sin \alpha} + \frac{BC}{\sin \beta} \cos s\right) = \frac{BC}{\sin \beta} \sin s \cos x$$

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$$\left(\frac{AB}{\sin\alpha} + \frac{BC}{\sin\beta}\cos s\right)\frac{\sin\beta}{BC\sin s} = \cot x$$

$$\frac{AB\sin\beta}{BC\sin\alpha\sin s} + \frac{BC\cos s\sin\beta}{BC\sin s\sin\beta} = \cot x$$

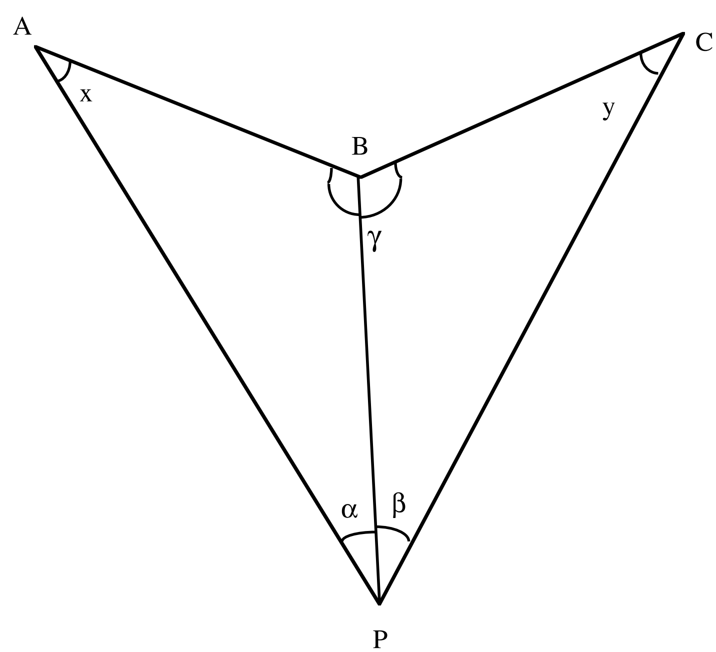
$$\frac{AB\sin\beta}{BC\sin\alpha\sin s} + \cot s = \cot x \qquad \text{[this is the equation solved first]}$$

$$y = s - x$$

With x and y determined, the sides AP, BP and CP can be calculated and hence the co-ordinates of P, as follows:

The azimuth of BP (Az_{BP}) can be determined using $Az_{BP} = Az_{AB} + \alpha + x$

The length of BP can be determined using $BP = \dfrac{AB\sin x}{\sin\alpha}$



Knowing the co-ordinates of B, Az_{BP} and BP, the co-ordinates of P can be easily computed. As a check, the equivalent solution can be obtain through the sides AP or CP, or using the angle y. Note that if P is close the danger circle, a solution will still be obtained, but the sum of $\alpha + \beta + \gamma$ will be close to 180°, probably in the range 175° to 185°. In this case, the solution will be highly sensitive to changes in α and β. If the solution is close to the danger circle, recomputed with the angles changed by about their precision and see how much the resulting co-ordinates change. It can be quite surprising! To facilitate this, press GTO R035, then R/S, to run the program with the same known points, but you can enter different observed angles.

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Azimuths in HP notation are used. Arbitrary co-ordinates are satisfactory. Plane surveying assumptions apply. The program uses no error checking on entered data. A check is made by showing the sum $\alpha + \beta + \gamma$. If this is close to 180°, the unknown point lies close to the danger circle and the result is highly suspect.

Sample Computation 1

Known Points

Point Name	X	Y
Point A	–25.336	778.136
Point B	–27.465	1179.927
Point C	–30.297	1555.643

Angles

Left (α) = 136° 35' 26"

Right (β) = 27° 19' 24"

Results

Unknown Point (P) X Co-ordinate = 26.009

Unknown Point (P) Y Co-ordinate = 1101.818

Check Angle = 344° 02' 32"

Sample Computation 2

Known Points

Point Name	X	Y
Point A	133.639	1548.712
Point B	158.065	1492.276
Point C	150.267	1353.056

Angles

Left (α) = 5° 01' 48"

Right (β) = 3° 41' 29"

Results

Unknown Point (P) X Co-ordinate = 116.784

Unknown Point (P) Y Co-ordinate = 1,186.818

Check Angle = 162° 06' 44"

This is not the ideal arrangement for a resection, as the measured angles are quite small. But the program will still produce an acceptable result.

This example is provided because the other example has negative co-ordinates and this tends to increase the chances of incorrect data entry. It happened to me, twice!

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Running the Program

Press XEQ R ENTER

Calculator displays RESECTION briefly, so that you know you are running the correct program.

Prompt ENTER LEFT X briefly, then X?

Enter X Co-ordinate for left known point.

Press R/S.

Prompt ENTER LEFT Y briefly, then Y?

Enter Y Co-ordinate for left known point.

Press R/S.

Prompt ENTER MID X briefly, then X?

Enter X Co-ordinate for middle known point.

Press R/S.

Prompt ENTER MID Y briefly, then Y?

Enter Y Co-ordinate for middle known point.

Press R/S.

Prompt ENTER RIGHT X briefly, then X?

Enter X Co-ordinate for right known point.

Press R/S.

Prompt ENTER RIGHT Y briefly, then Y?

Enter Y Co-ordinate for right known point.

Press R/S.

** Prompt ENTER ALPHA briefly, then X?

Enter left angle (α) in HP notation.

Press R/S.

Prompt ENTER BETA briefly, then X?

Enter right angle (β) in HP notation.

Press R/S.

Calculator displays RUNNING while doing the calculations.

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Prompt UNKNOWN X briefly, then X=

X co-ordinate of unknown point (P) is displayed.

Press R/S.

Prompt UNKNOWN Y briefly, then Y=

Y co-ordinate of unknown point (P) is displayed.

Press R/S.

Prompt CHECK VALUE briefly.

Sum $\alpha + \beta + \gamma$ is displayed in lower line of display in HP notation.

Check that value is not too close to 180°. At least 5 ° away, preferably 15° or more away.

Press R/S to clear flags. Program ends.

If you want to re-run the program with the same fixed points but different angles, press GTO R035, then R/S, and the program will start from the step labeled ** above, prompting with ENTER ALPHA. Changing the angles by small amounts can give you a good idea of the reliability of the solution.

Storage Registers Used

A	Left known point – X co-ordinate
B	Left known point – Y co-ordinate
C	Middle known point – X co-ordinate
D	Middle known point – Y co-ordinate
E	Right known point – X co-ordinate
F	Right known point – Y co-ordinate
G	Left measured angle — alpha (α)
H	Right measured angle — beta (β)
I	Interior angle at Middle known point — gamma (γ)
J	Distance from middle point to the unknown point
K	Vector from middle to right point (complex number format)
L	Vector from middle to left point (complex number format)
P	X co-ordinate of unknown point
Q	Y co-ordinate of unknown point
S	s = x + y in decimal degrees
X	Initial inputs, then angle x, then X co-ordinate of unknown point
Y	Initial inputs, then azimuth from middle to unknown point, then Y co-ordinate of unknown point
Z	360

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Labels Used

Label **R** Length = 581 Checksum = 17B0

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

The program sets flag 10, too allow equations to be displayed as prompts, and at the end of the program, resets flag 10 to its previous setting. The program uses flag 1 to record the state of flag 10 before the program started.