Traverse Closure with Area Calculation and Co-ordinates and Calculation of Curve Areas

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Line	Instruction	Display	User Instructions
B001	LBL B		Press XEQ B ENTER
B002	CLSTK		
B003	SF 10		
B004	CF 3		
B005	TRAVERSE 2		(Key in using EQN RCL T, RCL R, etc.)
B006	PSE		
B007	USE COORDS 0-1	USE COORDS 0-1	(Key in using EQN RCL U, RCL S, etc.)
B008	PSE		
B009	INPUT C	C?	
B010	RCL C		
B011	x = 0?		
B012	GTO B024		
B013	ENTER NO	ENTER NO	(Key in using EQN RCL E, RCL N, etc.)
B014	PSE		
B015	INPUT N	N?	
B016	ENTER E0	ENTER E0	(Key in using EQN RCL E, RCL N, etc.)
B017	PSE		
B018	INPUT E	E?	
B019	RCL E		
B020	0 i 1		(Key in as 0, then i , then 1, press ENTER.)
B021	×		
B022	RCL+ N		
B023	STO F		
B024	360		
B025	STO B		
B026	CLSTK		
B027	STO P		
B028	STO Q		
B029	STO M		
B030	XEQ W001		
B031	STO U		
B032	FS? 3		
B033	GTO B086		
B034	STO T		
B035	STO+ P		
B036	ARG		
B037	RCL P		
B038	ARG		

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B039 - B040 SIN B041 STO S B042 RCL U B043 ABS B044 STO× S B045 RCL P B046 ABS B047 STO× S B048 RCL S B049 2 B050 ÷ B051 STO+ Q B052 1 B053 STO+ M B054 RCL C B055 x = 0? B056 GTO B074 B057 RCL P B058 RCL+ F B059 XEQ X001 B061 RCL Q B062 ABS B063 R↓ B064 STOP B065 RCL P B066 ARG B067 x < 0? B068 RCL P B066 ARG B070 RCL P B071 ABS			
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B060RCL MB061RCL QB062ABSB063R↓B063R↓B064STOPB065RCL PB066ARGB067 $x < 0$?B068RCL+ BB069→HMSB070RCL PB071ABSB072STOPB073GTO B030B074RCL QB075ABSB076RCL PB077ARGB078 $x < 0$?B079RCL+ BB080→HMSB081RCL PB083RCL MB084STOP			
B061RCL QB062ABSB063R↓B064STOPB065RCL PB066ARGB067 $x < 0$?B068RCL+ BB069→HMSB070RCL PB071ABSB072STOPB073GTO B030B074RCL QB075ABSB076RCL PB077ARGB078 $x < 0$?B079RCL+ BB080→HMSB081RCL PB083RCL MB084STOP			
B062 ABS B063 R↓ B064 STOP B065 RCL P B066 ARG B067 x < 0?			
B063 R↓ B064 STOP B065 RCL P B066 ARG B067 x < 0?		· · · · ·	
B064 STOP B065 RCL P B066 ARG B067 x < 0?			
B065 RCL P B066 ARG B067 x < 0?			
B066 ARG B067 x < 0?			
B067 x < 0?			
B068 RCL+ B B069 →HMS B070 RCL P B071 ABS B072 STOP B073 GTO B030 B074 RCL Q B075 ABS B076 RCL P B077 ARG B078 x < 0?			
B069 →HMS B070 RCL P B071 ABS B072 STOP B073 GTO B030 B074 RCL Q B075 ABS B076 RCL P B077 ARG B078 x < 0?			
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B071 ABS B072 STOP B073 GTO B030 B074 RCL Q B075 ABS B076 RCL P B077 ARG B078 $x < 0$? B079 RCL+ B B080 \rightarrow HMS B081 RCL P B082 ABS B083 RCL M B084 STOP			
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B075 ABS B076 RCL P B077 ARG B078 $x < 0$? B079 RCL+ B B080 \rightarrow HMS B081 RCL P B082 ABS B083 RCL M B084 STOP	B073	GTO B030	
B076 RCL P B077 ARG B078 $x < 0$? B079 RCL+ B B080 \rightarrow HMS B081 RCL P B082 ABS B083 RCL M B084 STOP		RCL Q	
B077 ARG B078 $x < 0$? B079 RCL+ B B080 \rightarrow HMS B081 RCL P B082 ABS B083 RCL M B084 STOP	B075		
B078 $x < 0$? B079 RCL+ B B080 \rightarrow HMS B081 RCL P B082 ABS B083 RCL M B084 STOP	B076		
$B079$ $RCL+ B$ $B080$ $\rightarrow HMS$ $B081$ $RCL P$ $B082$ ABS $B083$ $RCL M$ $B084$ $STOP$	B077	ARG	
$B080$ \rightarrow HMS $B081$ RCL P $B082$ ABS $B083$ RCL M $B084$ STOP	B078	x < 0?	
B081 RCL P B082 ABS B083 RCL M B084 STOP	B079	RCL+ B	
B082ABSB083RCL MB084STOP	B080	→HMS	
B083 RCL M B084 STOP	B081	RCL P	
B084 STOP	B082	ABS	
	B083	RCL M	
B085 GTO B030	B084	STOP	
	B085	GTO B030]

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B086	ARC CHORD	
B087	PSE	
B088	0	
B089	STO Z	
B090	CONCAVE 0-1	
B091	PSE	
B092	INPUT Z	
B093	1	1
B094	STO Y	
B095	TANGENT 0-1	
B096	PSE	
B097	INPUT Y	
B098	x = 0?	
B099	GTO B158	
B100	90	
B101	RCL T	
B102	ARG	1
B103	RCL U	
B104	ARG	
B105	_	
B105	ABS	
B100	$x \le y$?	
B107	GTO B112	
B109	RCL B	
B110	x <> y	
B110 B111		
B112	STO Н	
B112 B113	RCL U	
B113 B114	ABS	
B115	RCL H	
B115 B116	SIN	
B110 B117	÷	
B118	2	
B110 B119	<u></u>	
B119 B120	STO R	
B120 B121	x^2	
B121 B122	RCL H	
B122 B123	\rightarrow RAD	
B123 B124	RCL H	
	2 RCL H	
B125 B126	2 ×	-
B127	SIN 2	-
B128	2	1
B129	÷.	-
B130		
B131	X	-
B132	ABS	

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B133	STO S	
B134	RCL Z	
B135	x = 0?	
B136	GTO B140	
B137	RCL S	
B138	STO-Q	
B139	GTO B142	
B140	RCL S	
B141	STO+ Q	
B142	RADIUS =	
B143	PSE	
B144	VIEW R	
B145	ARC LENGTH =	
B146	PSE PSE	
B140 B147	RCL H	
B147 B148	\rightarrow RAD	
<u>В148</u> В149	\rightarrow RAD 2	
B149 B150		
B151	RCL× R	
B152	STO L	
B153	VIEW L	
B154	CLSTK	
B155	CF 3	
B156	RCL U	
B157	GTO B034	
B158	ENTER RADIUS	
B159	PSE	
B160	INPUT R	
B161	RCL U	
B162	ABS	
B163	2	
B164	÷	
B165	RCL÷ R	
B166	ASIN	
B167	STO H	
B168	→RAD	
B169	2	
B170	RCL× H	
B171	SIN	
B172	2	
B173	÷	
B174	_	
B175	RCL R	
B176	x^2	
B170 B177	×	
B178	STO S	
B178 B179	RCL Z	
D1/7	NUL L	

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B180	x = 0?	
B181	GTO B185	
B182	RCL S	
B183	STO- Q	
B184	GTO B187	
B185	RCL S	
B186	STO+ Q	
B187	ARC LENGTH =	
B188	PSE	
B189	RCL H	
B190	→RAD	
B191	2	
B192	×	
B193	RCL× R	
B194	STO L	
B195	VIEW L	
B196	CLSTK	
B197	CF 3	
B198	RCL U	
B199	GTO B034	

This program is designed to completely hide the complex number work that the calculator performs to compute the traverse. All values entered and returned are presented in much the same manner as with the HP-33S calculator and its predecessors.

This program is a development and extension of the Closure 1A program. It requires the use of a slightly modified version of the V program, labeled W.

Notes

- (1) Set the calculator into DEGREES mode (press MODE 1) before starting.
- (2) This is a general traverse closure program that computes the azimuth and distance, plus area, to each point around the traverse, together with co-ordinates, if desired. It also computes the misclosure of a closed traverse.
- (3) The program can also calculate the area between the chord and arc of a circular curve, and include this in the overall area. It can work with curves that meet the adjacent lines as tangents, or lines that intersect the arc at an angle. It can also work with areas that are added to or subtracted from the total area of the polygon, i.e., convex and concave curves.
- (4) This program uses the W program as a sub-routine for data entry, so Program W must be in the calculator as program W (or the XEQ W001 at line B030 changed to reflect the changed label). The W program allows entry of azimuths in D.MMSS format (HP notation) and distances, while converting them to the internal format. Program W is one of the HP-35s Utilities programs (Utility 5). It differs from the V program in that it check for negative distances, converts them to positive distances, and sets Flag 3 if it finds a negative distance. This is used in the main program to indicate that a chord has been entered.

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- (5) This program uses the X program as a sub-routine for co-ordinate return from the internal format, so program X must be in the calculator as program X (or the XEQ X001 at line B059 changed to reflect the label change). The X program allows co-ordinates to be returned from the internal processing format of the calculator. Program X is one of the HP-35s Utilities programs (Utility 4).
- (6) The program allows the user to choose if the co-ordinates of each point are to be calculated. The user is prompted with USE COORDS briefly, followed by the C? prompt. If co-ordinates are desired, key in 1, if not, key in 0, then press R/S to continue.
- (7) After each side (azimuth and distance) has been entered, the calculator produces the following output.
 - A. If the user has selected to use co-ordinates, the calculator has the following data on the stack. It will stop and display this information, with the number of sides entered in the display in line 2, and the Easting co-ordinate of the point in line 1.

Stack Register	Contents
Т	Area of the traverse thus far (including curves)
Z	Northing co-ordinate of current point
Y	Easting co-ordinate of current point
Х	Number of sides entered

The user can scroll through the stack, using the $R \downarrow$ key, and can perform any other operation of interest to the data on the stack. This information is stored in memory registers for use later in the program, so the stack may be changed and worked with as needed. The user can also continue without viewing the stack.

When the user presses R/S, the calculator takes the line to the current forward point from the starting point, and converts it into the distance (which is placed in line 2, the X register) and the azimuth in degrees, minutes and seconds (HP notation) in line 1 of the display (the Y register).

Stack Register	Contents
Т	
Z	
Y	Azimuth from start to current point (in D.MMSSss)
X	Distance from start to current point

When the user presses R/S again, the calculator prompts for the azimuth of the next side to be entered. The azimuth should be entered in HP notation (DDD.MMSSss).

B. If the user has selected not to show co-ordinates, the calculator has the following data on the stack. It will stop and display this information, with the number of

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sides entered in line 2 of the display (the X register) and the distance of the misclosure or the line connecting the starting point to the current point, in line 1 of the display (Y register).

Stack Register	Contents
Т	Area of the traverse thus far
Z	Azimuth of the line from the start to current point (in HP notation)
Y	Distance of the line from the start to the current point
X	Number of sides entered

By pressing the $R \downarrow$ key, the user can see the azimuth in register Z of the stack. Pressing $R \downarrow$ again will show the area to the current point.

If the rectangular components of the misclosure (or the line from the start to the current forward point) are needed, press RCL P, then XEQ X ENTER. The Easting component of the misclosure (ΔE) will be displayed in the X register (line 2), while the Northing component (ΔN) will be displayed in the Y register (line 1).

When the user presses R/S again, the calculator prompts for the azimuth of the next side.

- (8) Azimuths are entered and displayed by themselves in HP notation, i.e., DDD.MMSSss.
- (9) This program forms the basis of the two missing distances (2MD) program. Enter the known sides using this program to begin the 2MD computation process.
- (10) In order to display the prompts, this program sets Flag 10. However, the program never ends, because it is up to the user to decide when to stop and move control elsewhere. So the program never clears Flag 10. If you require Flag 10 to be clear, in order to process equations, you must do this manually.
- (11) The program uses Flag 3 to see if a chord has been entered as the current side. If so, the area between the chord and the arc is calculated and added to the area calculation. The program then returns and calculated the area inside the polygon thus far, using the entered chord. So when each new point is displayed, the current area includes that between chords and arcs up to that point.
- (12) The program will work with curves that meet the straight line at the start of the curve as a tangent without additional work, except indicating that an arc has been entered. This is done by entering a negative distance for the side being entered. The user is prompted about whether the curve is convex or concave, and whether the curve and the starting point meet as tangents.
- (13) If the curve and the line at the starting point do not meet as tangents, the user is prompted to enter the radius of the curve, and calculation proceeds.
- (14) The program calculates the arc length of the curve, as well as the radius if it was a 'tangent' curve. Note that the relationship (tangent or not) between the curve and the adjacent line at the end of the curve is irrelevant to the calculation of the

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area, etc.; it is only the relationship between the curve and the line at the starting point that matters.

Theory

The traverse closure programs works by converting the entered azimuths (in DDD.MMSS, or HP notation) and distances into complex numbers (which act as 2-D vectors), which are then added to compute the location of points around the traverse. The area is computed by triangles developed by each new side of the traverse and the line from the starting point to the current forward point, and is updated with each new side. So the area is that of the polygon formed by the traverse entered thus far and the line from the start to the current point. This allows areas to be incremented for lot splitting calculations.

The azimuth and distance of the line from the start to the current point is also placed on the stack after each line. This allows a connecting line to be computed easily between two points. The final azimuth and distance thus shown is the traverse misclosure and the area is that of the traverse.

If the user chooses, the co-ordinates of the starting point may be entered, and if this choice is made, the calculator displays the co-ordinates of each point, in addition to the other information.

An arbitrary azimuth is satisfactory. Plane surveying assumptions apply. The program uses no error checking on entered data.

Dealing with Curves

When the program detects the presence of a chord, rather than a simple straight (indicated by entering a negative value for the distance of the chord), the program then prompts for whether or not the curve is concave or convex (convex is the default) with respect to the polygon, and whether or not the line entered as the side immediately before the chord formed a tangent to the curve (the default case is a tangent).

If the curve is convex, its area is added to the total being accumulated; if it is concave, the area is subtracted from the running total. This means that the area accumulated to the forward point (in this case, the end of the curve) is correct at the end of each line calculation.

If the previous side was a tangent to the curve, the program computes all the information needed from the deflection of the chord from the tangent and the tangent length. It displays the radius as a check (this is usually known), as well as the arc length. If the previous line was not a tangent to the curve, the program prompts for the radius, and computes the area. It also displays the arc length of the curve.

Once the area between the chord and the arc has been computed, the program computes the next line in the traverse, using the chord data, in the usual way. Note that it is irrelevant as to whether or not the line after the chord is tangential to the curve. This means that you can avoid having to enter the radius of a curve if only one end is not tangential, by computing around the traverse so that the tangential end comes first.

Tangent Case

In the case where the line coming into the curve (shown in black in Figure 1) is a tangent to the curve, the deflection of the chord (shown in blue in Figure 1) from that tangent is equal to half the deflection angle of the whole curve (θ), i.e., the angle at the center of the arc (where the radii meet). The radius of the curve can be calculated using the deflection angle and length of the chord, d, thus:

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$$r = \frac{\left(\frac{d}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}$$

The area of the segment of the circle between the chord and the arc (shown in red in Figure 1), A, can be calculated using:

$$A = \frac{\theta}{2}r^2 - \frac{1}{2}r^2\sin\theta = \left(\frac{\theta}{2} - \frac{\sin\theta}{2}\right)r^2$$

where θ indicates the size of the angle in radians.

The arc length, a, can be calculated using:

$$a = r \theta$$

where θ is in radians.

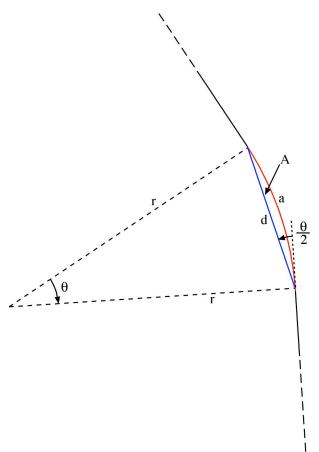


Figure 1 Diagram of the case where the line coming into the curve is a tangent to the curve. Note that the state of tangency at the end of the curve had no affect on the calculations.

HP-35S Calculator Program Closure 5A Traverse Closure with Area Calculation and Co-ordinates and Calculation of Curve Areas

The Non-Tangent Case

While it is more usual for curves to connect to other lines at tangent points, this tends to be for a large parcel, In many cases, a curved section of road forms a boundary to several parcels, and the boundary lines the separate these parcels meet the curve at other angles and are not tangential.

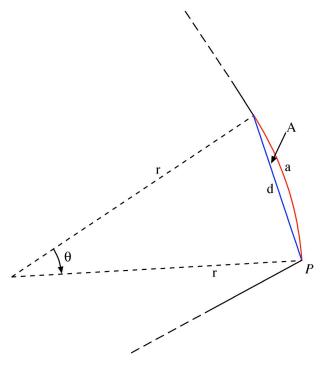


Figure 2 Diagram of the non-tangent case, where the line coming into the curve at point P is not tangential to the curve.

In Figure 2, the line that meets the curve at point P is clearly not a tangent to the curve. Therefore the deflection angle is no help in the calculations. All that is needed for calculating the area and the arc length are the radius (r) and the deflection angle of the curve (θ). So the user must supply the radius of the curve at this point in the calculation.

Knowing the radius and the chord length, half the deflection angle can be calculated thus:

$$\frac{\theta}{2} = \arcsin\left(\frac{\frac{d}{2}}{r}\right)$$

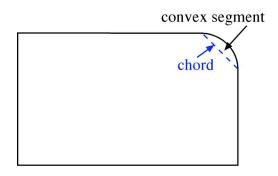
The segment area (A) and arc length (a) calculations can now be done using the same formulae as given for the tangent case.

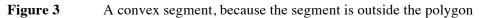
Convex and Concave

It is important to know whether the area of the segment between the chord and the arc should be added to the area of the polygon derived using the chord, or subtracted. This depends upon whether the segment is part of a concave or convex structure.

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If the segment is outside the polygon of which the chord is a side, it is the convex case. If the segment is inside the polygon, then it is the concave case.





In Figure 3, the segment is outside the five-sided polygon formed by the chord and the other sides. The area of the segment must therefore be added to the area of the polygon. The convex case is probably more common, so it is set up to be the default in the program.

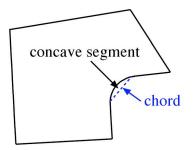


Figure 4 A concave segment, because the segment is inside the polygon.

In Figure 4, the segment is inside the seven-sided polygon formed by the chord and the other six sides. The area of the segment must therefore be subtracted from the area of the polygon.

Sample Computations

1. Without Curves

Azimuth	Distance
6° 53' 10"	72.00
112° 37' 20"	102.23
185° 39' 50"	29.04
181° 30' 00"	27.88
283° 54' 30"	102.38

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Final Results	DE	=	0.0228
	DN	=	-0.0022
	Misclosure Length	=	0.0229
	Misclosure Azimuth	=	95° 24' 15"
	Area	=	6,378.4660

Stepping through the Calculation

a. Without Co-ordinates

Press XEQ B ENTER

Calculator notes that this is the TRAVERSE 2 program.

Calculator prompts with USE COORDS 0–1, then C?

Key in 0, then press R/S.

Side 1

Calculator prompts with A? for azimuth of side.

Key in 6.5310, press R/S.

Calculator prompts with D? for distance of side.

Key in 72.00, press R/S.

Display shows:	72.0000	(distance from start)
	1.0000	(number of sides entered)

Press the $R \downarrow$ key twice, and the display becomes:

0.0000	(area thus far)
6.5310	(azimuth from start in HP notation (D.MMSS))

Press R/S.

Side 2

Calculator prompts with A? for azimuth of side.

Key in 112.372, press R/S.

Calculator prompts with D? for distance of side.

Key in 102.23, press R/S.

Display shows:	107.9004	(distance from start)
	2.0000	(number of sides entered)

Press the $R \downarrow$ key twice, and the display becomes:

3,542.3468	(area thus far)
72.3939	(azimuth from start in HP notation (D.MMSS))

Press R/S.

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Side 3

Calculator prompts with A? for azimuth of side.

Key in 185.395, press R/S.

Calculator prompts with D? for distance of side.

Key in 29.04, press R/S.

Display shows:	100.1841	(distance from start)
	3.0000	(number of sides entered)

Press the $R \downarrow$ key twice, and the display becomes:

4,984.4807(area thus far)88.0808(azimuth from start in HP notation (D.MMSS))

Press R/S.

Side 4

Calculator prompts with A? for azimuth of side.

Key in 181.3, press R/S.

Calculator prompts with D? for distance of side.

Key in 27.88, press R/S.

Display shows:	102.4027	(distance from start)
	4.0000	(number of sides entered)

Press the $R \downarrow$ key twice, and the display becomes:

6378.6396	(area thus far)
103.5423	(azimuth from start in HP notation (D.MMSS))

Press R/S.

Side 5

Calculator prompts with A? for azimuth of side.

Key in 283.543, press R/S.

Calculator prompts with D? for distance of side.

Key in 102.38, press R/S.

Display shows:	0.0229	(distance from start, also linear misclosure)
	5.0000	(number of sides entered)

Press the $R \downarrow$ key twice, and the display becomes:

6378.4660 (area thus far)95.2415 (azimuth from start, also azimuth of misclosure)

Pressing RCL P, then XEQ X ENTER brings the misclosure to the display in rectangular form. The Northing component of the misclosure (ΔN) is in the Y register (line 1) and is -0.0022, while the Easting component of the misclosure (ΔE) is in the X register (line 2) and is 0.0228.

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b. Using Co-ordinates Press XEQ B ENTER Calculator notes that this is the TRAVERSE 2 program. Calculator prompts with USE COORDS 0-1 briefly, then C? Key in 1, then press R/S. Calculator prompts ENTER N0 briefly, then N? Key in 1000.000, press R/S. Calculator prompts ENTER E0 briefly, then E? Key in 500.000, press R/S. Side 1 Calculator prompts with A? for azimuth of side. Key in 6.5310, press R/S. Calculator prompts with D? for distance of side. Key in 72.00, press R/S. Display shows: 508.6325 (Easting co-ordinate of current forward point) 1.0000 (Number of sides entered) Press the $R \downarrow$ key twice, and the display becomes: 0.0000 (Area thus far) 1.071.4806 (Northing co-ordinate of current forward point) Press R/S. (Azimuth for current forward point from starting point) Display shows: 6.5310 72.0000 (Distance from starting point to current forward point) Press R/S. Side 2 Calculator prompts with A? for azimuth of side. Key in 112.372, press R/S. Calculator prompts with D? for distance of side. Key in 102.23, press R/S. Display shows: 602.9971 (Easting co-ordinate of current forward point) 2.0000 (Number of sides entered) Press the $R\downarrow$ key twice, and the display becomes: 3,542.3468 (Area thus far) 1,032.1575 (Northing co-ordinate of current forward point)

Press R/S.

HP-35S Calculator	e	Closure 5A
		Area Calculation and Co-ordinates
and Calculati Display shows:	on of Curv 72.3939	(Azimuth for current forward point from starting point)
1 5	107.9004	(Distance from starting point to current forward point)
Press R/S.		
Side 3		
Calculator prompts with	th A? for azimut	h of side.
Key in 185.395, press	R/S.	
Calculator prompts with	th D? for distanc	e of side.
Key in 29.04, press R/S	S.	
Display shows:	600.1310 3.0000	(Easting co-ordinate of current forward point) (Number of sides entered)
Press the $R\downarrow$ key twice	, and the display	v becomes:
	4,984.4807 1,003.2593	(Area thus far) (Northing co-ordinate of current forward point)
Press R/S.		
Display shows:	88.0808 100.1841	(Azimuth for current forward point from starting point) (Distance from starting point to current forward point)
Press R/S.		
Side 4		
Calculator prompts with	th A? for azimut	h of side.
Key in 181.3, press R/S	S.	
Calculator prompts with	th D? for distanc	e of side.
Key in 27.88, press R/S	S.	
Display shows:	599.4012 4.0000	(Easting co-ordinate of current forward point) (Number of sides entered)
Press the $R\downarrow$ key, and	the display beco	mes:
	6,378.6396 975.3888	(Area thus far) (Northing co-ordinate of current forward point)
Press R/S.		
Display shows:	103.5423 102.4027	(Azimuth for current forward point from starting point) (Distance from starting point to current forward point)
Press R/S.		
Side 5		
Calculator prompts wit	th A? for azimut	h of side.
Key in 283.543, press 2	R/S.	
Calculator prompts with	th D? for distanc	e of side.
Key in 102.38, press R	/S.	

Traverse Cl	osure with	Area Calculation and Co-ordinates
and Calcula	tion of Cur	ve Areas
Display shows:	500.0228	(Easting co-ordinate of current forward point)
	5.0000	(Number of sides entered)
Press the $R\downarrow$ key, as	nd the display bec	omes:
	6,378.4660	(Area thus far)
	999.9978	(Northing co-ordinate of current forward point)
Press R/S.		
Display shows:	95.2415	(Azimuth for current forward point from starting point)
	0.0229	(Distance from starting point to current forward point)

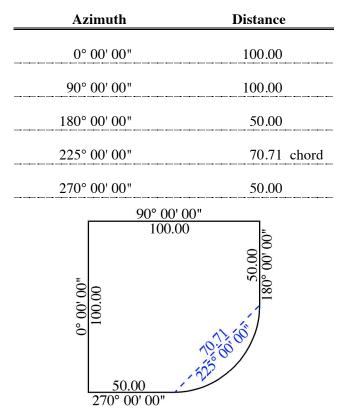
Closure 5A

Press RCL P, then XEQ X ENTER to bring the misclosure onto the stack in rectangular mode. The misclosure in Northings (latitude, ΔN) will be displayed in the Y register (line 1) as -0.0022. The misclosure in Eastings (departure, ΔE) will be displayed in the X register (line 2) and is 0.0228.

2. With Curves

a. Convex Tangent Case

In this example, the situation of using a curve that is tangential to the adjacent lines is used. The output for not using co-ordinates is provided.





Polygon with a convex curve that is tangential to the adjacent polygon lines.

Traverse Closure with Area Calculation and Co-ordinates and Calculation of Curve Areas

Press XEQ B ENTER

Calculator notes that this is the TRAVERSE 2 program.

Calculator prompts with USE COORDS 0–1, then C?

Key in 0, then press R/S.

Side 1

Calculator prompts with A? for azimuth of side.

Key in 0.0000, press R/S.

Calculator prompts with D? for distance of side.

Key in 100.00, press R/S.

Display shows:	100.0000	(distance from start)
	1.0000	(number of sides entered)

Press the $R \downarrow$ key twice, and the display becomes:

0.0000	(area thus far)
0.0000	(azimuth from start in HP notation (D.MMSS))

Press R/S.

Side 2

Calculator prompts with A? for azimuth of side.

Key in 90.0000, press R/S.

Calculator prompts with D? for distance of side.

Key in 100.00, press R/S.

Display shows:	141.4214	(distance from start)
	2.0000	(number of sides entered)

Press the $R \downarrow$ key twice, and the display becomes:

5,000.0000	(area thus far)
45.0000	(azimuth from start in HP notation (D.MMSS))

Press R/S.

Side 3

Calculator prompts with A? for azimuth of side.

Key in 180.0000, press R/S.

Calculator prompts with D? for distance of side.

Key in 50.00, press R/S.

Display shows:	111.8034	(distance from start)
	3.0000	(number of sides entered)

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Press the $R\downarrow$ key twice, and the display becomes:

7,500.0000(area thus far)63.2606(azimuth from start in HP notation (D.MMSS))

Press R/S.

Side 4, the chord

Calculator prompts with A? for azimuth of side.

Key in 225.0000, press R/S.

Calculator prompts with D? for distance of side.

Key in -70.71, press R/S. (Key in the number, then press the +/- key)

The calculator briefly display ARC CHORD to indicate that the side entered was a chord, and the '3' annunciator in the display should come on.

The calculator briefly displays CONCAVE 0–1, then prompts with Z?

As this is a convex curve, key in 0 and press R/S, or just press R/S as the 0 is already there. (0 means 'no' as the response to the question of concavity.)

The calculator briefly displays TANGENT 0-1, then prompts with Y?

As the last side is a tangent to the curve, key in 1 and press R/S, or just press R/S as the 1 is already there. (1 means 'yes' as the response to the question about the previous side being a tangent.)

The calculator briefly displays RADIUS =.

Display shows: R =

49.9995

Press R/S.

The calculator briefly displays ARC LENGTH =.

Display shows: L =

78.5391

Press R/S.

The '3' annunciator in the display should go out.

Display shows:	50.0005	(distance from start)
	4.0000	(number of sides entered)

Press the $R\downarrow$ key twice, and the display becomes:

9,463.4697	(area thus far)
89.5958	(azimuth from start in HP notation (D.MMSS))

Press R/S.

Traverse Closure with Area Calculation and Co-ordinates and Calculation of Curve Areas

Side 5

Calculator prompts with A? for azimuth of side.

Key in 270.0000, press R/S.

Calculator prompts with D? for distance of side.

Key in 50.00, press R/S.

Display shows:	0.0007	(distance from start)
	5.0000	(number of sides entered)

Press the $R \downarrow$ key twice, and the display becomes:

9,463.4577	(area thus far)
45.0000	(azimuth from start in HP notation (D.MMSS))

Showing the Misclosure

Press RCL P, then XEQ X ENTER to bring the misclosure onto the stack in rectangular mode. The misclosure in Northings (latitude, ΔN) will be displayed in the Y register (line 1) as 0.0005. The misclosure in Eastings (departure, ΔE) will be displayed in the X register (line 2) and is 0.0005.

The process of computing around the traverse and computing co-ordinates is almost exactly the same, except an addition R/S is needed during the display of the co-ordinate data.

b. Concave Non-Tangent Case

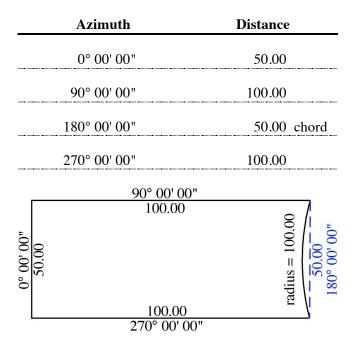


Figure 6 Polygon with a concave side that is not tangential to the adjacent sides.

In this example, co-ordinates are not displayed. The difference between the two processes are that an extra press of R/S is needed when displaying co-ordinates.

The radius of the curve is 100.00.

Traverse Closure with Area Calculation and Co-ordinates and Calculation of Curve Areas

Press XEQ B ENTER

Calculator notes that this is the TRAVERSE 2 program.

Calculator prompts with USE COORDS 0–1, then C?

Key in 0, then press R/S.

Side 1

Calculator prompts with A? for azimuth of side.

Key in 0.0000, press R/S.

Calculator prompts with D? for distance of side.

Key in 50.00, press R/S.

Display shows:	50.0000	(distance from start)
	1.0000	(number of sides entered)

Press the $R \downarrow$ key twice, and the display becomes:

0.0000	(area thus far)
0.0000	(azimuth from start in HP notation (D.MMSS))

Press R/S.

Side 2

Calculator prompts with A? for azimuth of side.

Key in 90.0000, press R/S.

Calculator prompts with D? for distance of side.

Key in 100.00, press R/S.

Display shows:	111.8034	(distance from start)
	2.0000	(number of sides entered)

Press the $R\downarrow$ key twice, and the display becomes:

2,500.0000	(area thus far)
63.2606	(azimuth from start in HP notation (D.MMSS))

Press R/S.

Side 3, the chord

Calculator prompts with A? for azimuth of side.

Key in 180.0000, press R/S.

Calculator prompts with D? for distance of side.

Key in -50.00, press R/S.

The calculator briefly displays ARC CHORD to show that the last side entered was a chord.

The calculator briefly displays CONCAVE 0–1, then prompts with Z?

In this case, the curve is concave, so key in 1 (for 'yes') and press R/S.

Traverse Closure with Area Calculation and Co-ordinates and Calculation of Curve Areas

The calculator briefly displays TANGENT 0-1, then prompts with Y?

In this case, the previously entered line is not tangential to the curve, so key in 0 (for 'no') and press R/S.

The calculator briefly displays ENTER RADIUS, then prompts with R?

Key in 100.00 and press R/S.

The calculator briefly displays ARC LENGTH =

Display shows: L =

50.5361

Press R/S.

Display shows:	100.0000	(distance from start)
	3.0000	(number of sides entered)

Press the $R \downarrow$ key twice, and the display becomes:

4,893.8120	(area thus far)
90.0000	(azimuth from start in HP notation (D.MMSS))

Press R/S.

Side 4

Calculator prompts with A? for azimuth of side.

Key in 270.0000, press R/S.

Calculator prompts with D? for distance of side.

Key in 100.00, press R/S.

Display shows:	0.0000	(distance from start)
	4.0000	(number of sides entered)

Press the $R \downarrow$ key twice, and the display becomes:

4,893.8120	(area thus far)
0.0000	(azimuth from start in HP notation (D.MMSS))

Showing the Misclosure

Press RCL P, then XEQ X ENTER to bring the misclosure onto the stack in rectangular mode. The misclosure in Northings (latitude, ΔN) will be displayed in the Y register (line 1) as 0.0000. The misclosure in Eastings (departure, ΔE) will be displayed in the X register (line 2) and is 0.0000.

Storage Registers Used

- **A** Used by the W program for the entered azimuth.
- **B** Stores 360 for azimuth correction.
- **C** Test for displaying co-ordinates: 1 = YES; 0 = NO.
- **D** Used by the W program for the entered distance.
- **E** Easting co-ordinates of the starting point.

Traverse Closure with Area Calculation and Co-ordinates and Calculation of Curve Areas

- **F** Co-ordinates of starting point, as a complex number.
- **H** Half angle at center of arc (deflection angle, θ)
- **I** Used by the W and X programs to address the additional storage registers.
- L Arc length
- **M** The number of sides entered.
- **N** The Northing co-ordinate of the starting point.
- **P** Current position of forward point, as a complex number.
- **Q** Current area.
- **R** Radius of curve
- **S** Temporary storage for area calculation.
- T Side before the last entered
- U Last side entered, as a complex number.
- **Y** Test for tangent to curve: 1 = YES (default); 0 = NO.
- **Z** Test for convex or concave curve: 0 = CONVEX (default), 1 = CONCAVE.

Statistical Registers: not used.

Other registers: 10, 11 and 12 used by the W program; 10 used by the X program.

Labels Used

Label **B** Length = 732 Checksum = B9E5

Use the Length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.

Routines Called

The program labeled W, which takes an azimuth in degrees, minutes and seconds (in HP notation), and a distance, and converts them to a complex number for processing in the calculator. This routines uses storage location A and D, but copies out and replaces the contents of these storage locations in order to preserve them. It also uses storage register I for indirect addressing, which uses registers 10, 11 and 12.

The program labeled X, which takes a complex number and converts it to the rectangular components, is called for co-ordinate presentation in this program. It uses storage register I for indirect addressing, which uses register 10.

Label WLength = 146Checksum = A1A3Label XLength = 53Checksum = C46D

HP-35S Calculator Program Closure 5A **Traverse Closure with Area Calculation and Co-ordinates and Calculation of Curve Areas** Flags Used

The program sets Flag 10 to allow display of equations as prompts. Because the program never comes to a defined end (this is up to the user to decide), Flag 10 is never reset to its previous value. The user will have to do this if it is required.

The program uses Flag 3 to indicate that a chord has been entered, and so to calculate the segment area. This is done by noting the presence of a negative distance entered for the chord. Flag 3 is reset once the curve calculation has been completed. The state of Flag 3 is indicated in the display, with s small 3 at the top of the display turned on when the flag is set.