HP-35s Calculator Program — Closure 7A

Traverse Program using Latitude and Longitude and the Gauss Mid-Latitude Formulae

Programmer: Dr. Bill Hazelton

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Line	Instruction	Display	User Programming Instructions
J001	LBL J		LBL J
J002	CLSTK		CLEAR 5
J003	FS? 10		FLAGS 3 .0
J004	GTO J008		
J005	SF 1		FLAGS 1 1
J006	SF 10		FLAGS 1 .0
J007	GTO J009		
J008	CF 1		FLAGS 2 1
J009	GAUSS M–L TRAV		(Key in using EQN RCL G, RCL A, etc.)
J010	PSE		PSE
J011	6378137		
J012	STO A		STO A
J013	6.699438 E–3		
J014	STO E		STO E
J015	CHECK–ENTER A		(Key in using EQN RCL C, RCL H, etc.)
J016	PSE		PSE
J017	INPUT A		INPUT A
J018	CHECK–ENTER E		(Key in using EQN RCL C, RCL H, etc.)
J019	PSE		PSE
J020	INPUT E		INPUT E
J021	1		
J022	STO M		STO M
J023	CHOOSE UNITS		(Key in using EQN RCL C, RCL H, etc.)
J024	PSE		PSE
J025	FEET–METERS		(Key in using EQN RCL F, RCL E, etc.)
J026	PSE		PSE
J027	INPUT M		INPUT M
J028	0		
J029	STO H		STO H
J030	STO F		STO F
J031	STO L		STO L
J032	AV HT ALL LINES		(Key in using EQN RCL A, RCL V, etc.)
J033	PSE		PSE
J034	ENTER HT		(Key in using EQN RCL E, RCL N, etc.)
J035	PSE		PSE
J036	INPUT H		INPUT H
J037	START POSN		(Key in using EQN RCL S, RCL T, etc.)
J038	PSE		PSE
J039	ENTER LAT		(Key in using EQN RCL E, RCL N, etc.)

Traverse Closure Using Latitude and Longitude

Line	Instruction	Display	User Programming Instructions
J040	PSE		↱ PSE
J041	INPUT F		↰ INPUT F
J042	ENTER LONG		(Key in using EQN RCL E, RCL N, etc.)
J043	PSE		↱ PSE
J044	INPUT L		↰ INPUT L
J045	RCL F		
J046	HMS→		↰ HMS→
J047	STO F		↱ STO F
J048	STO Y		↱ STO Y
J049	RCL L		
J050	HMS→		↰ HMS→
J051	STO L		↱ STO L
J052	STO X		↱ STO X
J053	1		
J054	RCL H		
J055	RCL× M		
J056	1.571 E–7		
J057	×		
J058	–		
J059	STO H		↱ STO H
J060	ENTER AZIMUTH		(Key in using EQN RCL E, RCL N, etc.)
J061	PSE		↱ PSE
J062	INPUT Z		↰ INPUT Z
J063	ENTER DISTANCE		(Key in using EQN RCL E, RCL N, etc.)
J064	PSE		↱ PSE
J065	INPUT D		↰ INPUT D
J066	RCL Z		
J067	HMS→		↰ HMS→
J068	STO Z		↱ STO Z
J069	RCL M		
J070	STO× D		↱ STO × D
J071	RCL H		
J072	STO× D		↱ STO × D
J073	RCL Z		
J074	SIN		
J075	RCL× D		
J076	RCL Z		
J077	COS		
J078	RCL× D		
J079	RCL÷ A		
J080	→ DEG		↱ →DEG
J081	STO S		↱ STO S
J082	2		
J083	÷		
J084	STO B		↱ STO B
J085	x < > y		

Traverse Closure Using Latitude and Longitude

Line	Instruction	Display	User Programming Instructions
J086	RCL F		
J087	RCL+ B		
J088	COS		
J089	÷		
J090	RCL÷ A		
J091	→ DEG		↱ →DEG
J092	STO T		↱ STO T
J093	2		
J094	÷		
J095	STO C		↱ STO C
J096	1.004		
J097	STO Q		↱ STO Q
J098	RCL F		
J099	RCL+ B		
J100	SIN		
J101	RCL C		
J102	TAN		
J103	×		
J104	RCL B		
J105	COS		
J106	÷		
J107	ATAN		↱ ATAN
J108	STO G		↱ STO G
J109	1		
J110	RCL– E		
J111	RCL× A		
J112	RCL F		
J113	RCL+ B		
J114	SIN		
J115	RCL× E		
J116	1		
J117	x <> y		
J118	–		
J119	1.5		
J120	y^x		
J121	÷		
J122	STO R		↱ STO R
J123	RCL A		
J124	1		
J125	RCL F		
J126	RCL+ B		
J127	SIN		
J128	RCL× E		
J129	–		
J130	√x		
J131	÷		

Line	Instruction	Display	User Programming Instructions
J132	STO N		STO N
J133	RCL D		
J134	RCL÷ R		
J135	RCL Z		
J136	RCL+ G		
J137	COS		
J138	×		
J139	→ DEG		→DEG
J140	STO S		STO S
J141	RCL Z		
J142	RCL+ G		
J143	SIN		
J144	RCL× D		
J145	RCL÷ N		
J146	RCL F		
J147	RCL+ B		
J148	COS		
J149	÷		
J150	→ DEG		→DEG
J151	STO T		STO T
J152	RCL S		
J153	2		
J154	÷		
J155	STO B		STO B
J156	RCL T		
J157	2		
J158	÷		
J159	STO C		STO C
J160	ISG Q		ISG Q
J161	GTO J098		
J162	RCL S		
J163	STO+ F		STO + F
J164	RCL T		
J165	STO+ L		STO + L
J166	RCL F		
J167	→ HMS		→HMS
J168	STO F		STO F
J169	CURRENT POINT		(Key in using EQN RCL C, RCL U, etc.)
J170	PSE		PSE
J171	LATITUDE		(Key in using EQN RCL L, RCL A, etc.)
J172	PSE		PSE
J173	VIEW F		VIEW F
J174	RCL L		
J175	→ HMS		→HMS
J176	STO L		STO L
J177	LONGITUDE		(Key in using EQN RCL L, RCL O, etc.)

Traverse Closure Using Latitude and Longitude

Line	Instruction	Display	User Programming Instructions
J178	PSE		↱ PSE
J179	VIEW L		↰ VIEW L
J180	RCL Z		
J181	RCL+ G		
J182	RCL+ G		
J183	→ HMS		↱ →HMS
J184	STO Q		↱ STO Q
J185	FWD AZIMUTH		(Key in using EQN RCL F, RCL W, etc.)
J186	PSE		↱ PSE
J187	VIEW Q		↰ VIEW Q
J188	RCL F		
J189	HMS→		↰ HMS→
J190	STO F		↱ STO F
J191	RCL L		
J192	HMS→		↰ HMS→
J193	STO L		↱ STO L
J194	NEW LINE (0–1)		(Key in using EQN RCL N, RCL E, etc.)
J195	PSE		↱ PSE
J196	1		
J197	STO J		↱ STO J
J198	INPUT J		↰ INPUT J
J199	RCL J		
J200	x > 0?		↱ x ? 0 4
J201	GTO J060		
J202	RCL F		
J203	RCL– Y		
J204	→ HMS		↱ →HMS
J205	STO B		↱ STO B
J206	RCL L		
J207	RCL– X		
J208	→ HMS		↱ →HMS
J209	STO C		↱ STO C
J210	MISCLOSURE		(Key in using EQN RCL M, RCL I, etc.)
J211	PSE		↱ PSE
J212	MISC LATITUDE		(Key in using EQN RCL M, RCL I, etc.)
J213	PSE		↱ PSE
J214	VIEW B		↰ VIEW B
J215	MISC LONG		(Key in using EQN RCL M, RCL I, etc.)
J216	PSE		↱ PSE
J217	VIEW C		↰ VIEW C
J218	PROGRAM END		(Key in using EQN RCL P, RCL R, etc.)
J219	PSE		↱ PSE
J220	FS? 1		↰ FLAGS 3 1
J221	CF 10		↰ FLAGS 2 .0
J222	RTN		↰ RTN

Notes

(1) Simple computation of co-ordinates around a traverse, together with a simple computations of misclosure, where the traverse point locations are latitudes and longitudes, and the lines have their starting azimuth and linear distance available.

(2) Brief prompts are provided before each requirement for data entry, as well as before results are displayed. Each prompt shows for about 1 second, and is then replaced by the value or request for input.

(3) Co-ordinates of the traverse points (latitude and longitude) are not stored, and so must be written down to record them.

(4) Angles, including latitudes, longitudes and azimuths, are entered and displayed in HP notation, i.e., DDD.MMSS. Internal storage of angles and azimuths is in decimal degrees.

(5) Latitudes are positive North of the Equator, negative South of the Equator. Longitudes are positive to the East of the Greenwich meridian, negative to the West of the Greenwich meridian. This means that in the US, latitudes are positive and longitudes are negative.

(5) The program computes the latitude and longitude of the next traverse point from the azimuth and distance of the line from the current point. This is the classical 'forward' line computation problem in geodesy. The program uses the Gauss Mid-Latitude Formulae for the calculation, iterating each line four times. The program can also be used for stand-alone Gauss Mid-Latitude computations.

(6) The forward azimuth of the line at the end of each line is displayed. This allows the user to compute the azimuth of the next line, using the back azimuth of the current line and the angle measured at the traverse point, if this is required.

(7) The Gauss Mid-Latitude Formulae take care of any spherical excess in the figure, leaving the measured angle misclosure in the resulting azimuths.

(8) The formulae are designed to work with the WGS84/NAD83/GRS80 ellipsoid. If a different ellipsoid is required, the a and e^2 values can be changed at the start of the program. If computations on a spherical figure are required, enter the required radius for a, and set e^2 to zero. Values for some other ellipsoids are given later in this document.

(9) Users can enter an average height for lines, to allow lines to be reduced to the ellipsoid, but this is an average value for the entire traverse. If the lines have already been reduced to the ellipsoid, or this is not required, enter zero for the height.

(10) Users can elect to enter distances in feet by entering 0.3048 when prompted for FEET–METERS early in the program. To use meters, retain a value of 1, the program default. Other units can be used, if a conversion factor to meters is entered instead of the 0.3048. Long lines may need 0.30480061 for US Survey feet, while 0.3048 is used for International feet and approximate work in feet.

(11) The resulting misclosure is expressed in angular terms, in HP notation, and is the amount by which the final latitude and longitude miss the starting values. To convert these values to meters, multiply the number of seconds of latitude by 30, and the number of seconds of longitude by 30 cos ϕ. To get the values in feet multiply by 100 and 100 cos ϕ, respectively, instead. Note that these will be approximate.

Theory

This program uses the Gauss Mid-Latitude formulae to calculate the position of the point at the end of a line, given the starting position (in latitude, ϕ, and longitude, λ), the forward azimuth at the known point, and the distance (in either feet or meters). This is the classical 'forward' problem of geodetic line computation.

For the forward solution, the Gauss Mid-Latitude formulae require iteration to reach a solution, but are the simplest and quickest geodetic formulae for this type of task. The formulae are accurate to better than 0.001" in latitude and longitude (0.3 m, 0.1 ft), for worst-case lines up to 20 miles (32 km). If greater precision is required, use a different geodetic long-line formula (e.g., Robbins' or Rudoe's formulae).

Within the program, when a line's azimuth, θ, and distance, d, are first entered, they are converted to an initial, approximate latitude and longitude differences ($\Delta\phi_0$ and $\Delta\lambda_0$, respectively), using:

$$\Delta\phi_0 = \frac{d\cos\theta}{a} \qquad \Delta\lambda_0 = \frac{d\sin\theta}{a\cos\phi}$$

The mid-latitude of the line, ϕ_m, is computed using: $\phi_m = \phi + \frac{\Delta\phi}{2}$ [1]

The change in azimuth over the length of the line, $\Delta\theta$, is computed using:

$$\tan\frac{\Delta\theta}{2} = \tan\frac{\Delta\lambda}{2}\sin\phi_m\sec\frac{\Delta\phi}{2} \qquad [2]$$

The radii of curvature in the meridian and prime vertical at the mid-point of the line, ρ_m and ν_m, respectively, are calculated, using:

$$\rho_m = \frac{a(1-e^2)}{(1-e^2\sin^2\phi_m)^{\frac{3}{2}}} \qquad [3]$$

$$\nu_m = \frac{a}{\sqrt{(1-e^2\sin^2\phi_m)}} \qquad [4]$$

The differences in latitude and longitude are then calculated, using:

$$\Delta\phi = \frac{d}{\rho_m}\cos\left(\theta + \frac{\Delta\theta}{2}\right) \qquad [5]$$

$$\Delta\lambda = \frac{d}{\nu_m}\sin\left(\theta + \frac{\Delta\theta}{2}\right)\sec\phi_m \qquad [6]$$

The values from equations [5] and [6] are returned to equation [1], and the process re-iterated until the changes in latitude and longitude are too small to worry about. In most cases, three iterations are sufficient, but the program uses four iterations, just to be sure.

Traverse Closure Using Latitude and Longitude

Distances are converted to ellipsoidal distances using the average height (h_m) for the region (or single line) entered. Entering zero for the height means the height scale factor has no effect on line length. The formula used is as follows, which is good to about 1 in 10,000 if the height is good to within 60 meters. The ellipsoidal distance is equal to the 'horizontal' distance times the height scale factor.

$$\text{Height Scale Factor} = 1 - (h_m \times 0.1571 \times 10^{-6})$$

At the end of each line, the program displays the latitude and longitude of the end point, as well as the forward azimuth of the line at this point. For most lines of any significant length, this will differ from the forward azimuth at the start of the line. By converting this forward bearing to a back bearing (by adding or subtracting 180°), an angle measured at the end point can be used to obtain the forward azimuth of the next line. This is done manually by the user, and is not included in the program, as it is not something that will be needed by all users.

The program stores the initial point values, so that a comparison can be made at the end of a traverse, if desired. The difference is calculated and shown to the user.

Sample Computation

Traverse Data and Results

Point	Line	Azimuth	Distance	Latitude	Longitude
A				40° 02' 25".000	–83° 01' 25".000
	A–B	47° 51' 27"	14,302.785		
B				40° 07' 35".189	–82° 53' 57".427
	B–C	140° 32' 56"	12,821.076		
C				40° 02' 14".812	–82° 48' 14".056
	C–D	235° 28' 29"	15,093.269		
D				39° 57' 37".772	–82° 56' 57".579
	D–E	274° 28' 12"	6,394.974		
E				39° 57' 53".807	–83° 01' 26".012
	E–A	0° 09' 50"	8,383.815		
A				40° 02' 25".001	–83° 01' 25".001

Misclosure Latitude (ϕ) = 0° 00' 00".001 = 0".0007 = 0.021 m

Longitude (λ) = –0° 00' 00".001 = –0".0008 = –0.018 m

The angular misclosure around this figure was initially 2", of which about 1" was spherical excess. The linear misclosure when computed by other means is very close to the figures above.

Note that the misclosure is determined from very small differences at the least significant end of a long number, and so is affected by the limitations in the calculator's internal precision. Calculation of the same traverse using different equipment (e.g., a spreadsheet) should give the same results for locations and azimuths, but there may be some small differences in the misclosure.

Storage Registers Used

A	a = semi-major axis of ellipsoid = 6378137 m for WGS84/NAD83/GRS80
B	$\Delta\phi/2$ = half the latitude difference for the line
C	$\Delta\lambda/2$ = half the longitude difference for the line
D	Distance, i.e., length of the line
E	e^2 = eccentricity of ellipsoid = 0.006699438 for WGS84/NAD83/GRS80
F	ϕ, latitude of starting point of each line
G	$\Delta\theta/2$ = half the azimuth change for the line
H	Height above ellipsoid, then the height-scale factor for line lengths. By default, height = 0
J	Response variable for whether another line is to be processed
L	λ, longitude of starting point of each line
M	unit to meters conversion factor; by default 1.0 for meters
N	ν_m = radius of curvature of the ellipsoid in the prime vertical at the mid-point of the line
Q	counter for calculation loop (1.004 by default), then forward azimuth of line
R	ρ_m = radius of curvature of the ellipsoid in the meridian at the mid-point of the line
S	$\Delta\phi$, the difference in latitude for the line
T	$\Delta\lambda$, the difference in longitude for the line
X	λ_0, longitude of the initial point of the traverse
Y	ϕ_0, the latitude of the initial point of the traverse
Z	θ, azimuth at the start of the line

Labels Used

Label **J**	Length = 959	Checksum = D8B4

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after program entry.

Flags Used

Flags 1 and 10 are used by this program. Flag 10 is set for this program, so that equations can be shown as prompts. Flag 1 is used to record the setting of Flag 10 before the program begins. At the end of the program, Flag 10 is reset to its original value, based on the value in Flag 1.

Ellipsoidal Values

WGS84/NAD83/GRS80	a = 6 378 137 m	e^2 = 0.006 699 438 00
Clark 1866	a = 6 378 206.4 m	e^2 = 0.006 768 658
WGS72	a = 6 378 135 m	e^2 = 0.006 694 317 778
ANS (Australian)	a = 6 378 160 m	e^2 = 0.006 694 541 855

Running the Program

To start the program, press XEQ J, then press ENTER.

The calculator briefly displays GAUSS M–L TRAV, then briefly displays CHECK–ENTER A.

The calculator stops and prompts with A?

Key in a value for ellipsoid semi-major axis, or ignore to retain default value (the WGS84 value).

Press R/S to continue.

The calculator displays CHECK–ENTER E briefly, then stops and prompts with E?

Key in a value for ellipsoid eccentricity, or ignore to retain default value (the WGS84 value).

Press R/S to continue.

The calculator briefly displays CHOOSE UNITS, the briefly displays FEET-METERS, then stops and prompts with M?

Enter unit conversion value, or ignore to retain setting for distances in meters (value of 1). Enter 0.3048 for International feet; 0.30480061 for US Survey feet; 0.201168 for chains, etc.

Press R/S to continue.

The calculator briefly displays AV HT ALL LINES (although the S won't be visible), then briefly displays ENTER HT, then stops and prompts with H?

Enter average height above the ellipsoid for all lines to be processed. Ignore to retail default value of zero. Enter the height in the units you selected at the FEET–METERS prompt.

Press R/S to continue.

The calculator briefly displays START POSITION, then briefly displays ENTER LAT, then stops and prompts with F?

Enter the latitude (ϕ) of the starting point, in degrees, minutes and seconds, in HP notation (D.MMSSsss). Remember to include a negative sign, if in the southern hemisphere.

Press R/S to continue.

The calculator briefly displays ENTER LONG, then stops and prompts with L?

Enter the longitude (λ) of the starting point, in degrees, minutes and seconds, in HP notation (D.MMSSsss). Remember to include a negative sign, if in the western hemisphere.

Press R/S to continue.

Top of Loop Point

The calculator briefly displays ENTER AZIMUTH, then stops and prompts with Z?

Enter azimuth of the line at the starting point (θ) in HP notation.

Press R/S to continue.

Traverse Closure Using Latitude and Longitude

The calculator briefly displays ENTER DISTANCE, then stops and prompts with D?

Enter the length of the line in the units previous selected.

Press R/S to continue.

The calculator displays RUNNING for a while. Then the calculator briefly displays CURRENT POINT, the briefly displays LATITUDE, then stops and displays F= and the latitude of the far end of the line just entered, displayed in HP notation.

Press R/S to continue.

The calculator displays LONGITUDE briefly, then stops and displays L= and the longitude of the far end of the line just entered, displayed in HP notation.

Press R/S to continue.

The calculator displays FWD AZIMUTH briefly, then stops and displays Q= and the forward azimuth of the line at the far end of the line just entered, displayed in HP notation.

Press R/S to continue.

The calculator briefly displays NEW LINE (0–1), then stops and prompts with J? and the default value of 1. To go on to do the next line in the traverse, press R/S and the program will go to the *Top of Loop Point*, above. If all the sides have been entered, key in 0 and press R/S to calculate the misclosure.

The calculator displays MISCLOSURE briefly, the briefly displays MISC LATITUDE, then stops and displays B= and the misclosure in latitude (difference between start and end latitudes), displayed in HP notation.

Press R/S to continue.

The calculator briefly displays MISC LONGITUDE, then stops and shows C= and the misclosure in longitude (difference between start and end longitudes), displayed in HP notation.

Pressing R/S again will reset the flags, briefly display PROGRAM END, and end the program. If the program was called from another location, control will return to that point.

The misclosure in latitude will remain in the Y register, and the misclosure in longitude will remain in the X register, on the screen of the calculator. They can now be converted to whatever units interest you, after having converted them to decimal degrees using the HMS→ function.

Note that this misclosure is being determined from small differences at the least significant end of the calculator’s storage capability, and so will be approximate at best. It will give an idea of the degree of magnitude of the misclosure, rather than an exact amount.