



















**Roots of Complex Numbers**

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Mnemonic: R for Complex Roots

Line	Instruction	User Instructions
R001	LBL R	 LBL R
R002	CLSTK	 CLEAR 5
R003	FS? 10	 FLAGS 3 .0
R004	GTO R008	
R005	SF 1	 FLAGS 1 1
R006	SF 10	 FLAGS 1 .0
R007	GTO R009	
R008	CF 1	 FLAGS 2 1
R009	COMPLEX ROOT	(Key in using EQN RCL C RCL O, etc.)
R010	PSE	 PSE
R011	ENTER Z	(Key in using EQN RCL E RCL N, etc.)
R012	PSE	 PSE
R013	INPUT Z	 INPUT Z
R014	ENTER N	(Key in using EQN RCL E RCL N, etc.)
R015	PSE	 PSE
R016	RAD	MODE 2
R017	INPUT N	 INPUT N
R018	1	
R019	—	
R020	STO L	 STO L
R021	RCL Z	
R022	ABS	 ABS
R023	STO R	 STO R
R024	RCL Z	
R025	ARG	 ARG
R026	STO A	 STO A
R027	RCL R	
R028	RCL N	
R029	$x\sqrt{y}$	 $x\sqrt{y}$
R030	STO B	 STO B
R031	RCL A	
R032	RCL ÷ N	
R033	COS	
R034	RCL A	
R035	RCL ÷ N	
R036	SIN	
R037	i	[The i key, next to the cursor keys]
R038	×	
R039	+	

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Line	Instruction	User Instructions
R040	RCL× B	
R041	STO C	 STO C
R042	FIRST ROOT	(Key in using EQN RCL F RCL I, etc.,)
R043	PSE	 PSE
R044	VIEW C	 VIEW C
R045	RCL L	
R046	1	
R047	STO— L	 STO— L
R048	$\pi$	 $\pi$
R049	2	
R050	$\times$	
R051	STO+ A	 STO+ A
R052	RCL A	
R053	RCL÷ N	
R054	COS	
R055	RCL A	
R056	RCL÷ N	
R057	SIN	
R058	<b>i</b>	[The <b>i</b> key, next to the cursor keys]
R059	$\times$	
R060	+	
R061	RCL× B	
R062	STO D	 STO D
R063	NEXT ROOT	(Key in using EQN RCL N RCL E, etc.)
R064	PSE	 PSE
R065	VIEW D	 VIEW D
R066	RCL L	
R067	$x > 0?$	 $x > 0?$
R068	GTO R045	
R069	PROGRAM END	(Key in using EQN RCL P RCL R, etc.)
R070	PSE	 PSE
R071	FS? 1	 FLAGS 3 1
R072	CF 10	 FLAGS 2 .0
R073	DEG	MODE 1
R074	RTN	 RTN

**Notes**

- (1) A simple program to calculate the roots of a complex number using De Moivre's formula.
- (2) The user provides the complex number using the standard calculator entry methods, i.e.,  $\Re i \Im$ , or  $r \theta a$ . The 'number' of the roots to be calculated is then entered.
- (3) The results (the various roots) are shown as complex numbers in the current complex display mode of the calculator. If the display is in polar complex display mode, the displayed angles (the value after the  $\theta$ ) will be in radians.

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- (4) The program works largely in radians mode, but returns to degrees mode at the end of the program. If the program is stopped before coming to the end, the calculator may remain in radians mode.
- (5) The program sets Flag 10, so that equations can be displayed as prompts. The program completes by returning Flag 10 to its setting when the program began.

**Theory**

De Moivre's formula can be written as:

$$(\cos x + i \sin x)^n = \cos(nx) + i \sin(nx)$$

The formula can be used to find the  $n$ th roots of a complex number, as follows. If  $z$  is a complex number, written in polar form as:

$$z = r(\cos x + i \sin x)$$

then:

$$z^{\frac{1}{n}} = [r(\cos x + i \sin x)]^{\frac{1}{n}} = r^{\frac{1}{n}} \left[ \cos\left(\frac{x + 2k\pi}{n}\right) + i \sin\left(\frac{x + 2k\pi}{n}\right) \right]$$

where  $k$  is an integer, and to get the  $n$  different roots of  $z$  one only needs to consider values of  $k$  from 0 to  $n-1$ .

**Running the Program**

Press XEQ R, then press ENTER.

Program displays COMPLEX ROOTS briefly, then ENTER Z briefly, then prompts:

Z?

Key in the complex number, or if using a real number, key it in as a complex number, then press R/S. For example, to find the roots of the real number +1, key in 1 i 0, then press R/S.

Program displays ENTER N briefly, then prompts:

N?

Key in the value of  $n$ , the integer number of the roots to be obtained (i.e., finding the  $n$ th root). Press R/S. For example, key in 8 and press R/S/

The calculator briefly displays RUNNING, then briefly displays FIRST ROOT, then stops and shows the first root calculated as a complex number. In this case, if the calculator is in rectangular complex number display mode, the display will show:

C=  
1.0000 i 0.0000

The first root of +1 is, simply, 1. Press R/S to continue.

**Roots of Complex Numbers**

The calculator briefly displays RUNNING, then briefly displays NEXT ROOT, then stops and shows the next root calculated as a complex number. The display will show (if in rectangular mode):

$$C= \\ 0.7071 \text{ i } 0.7071$$

This is equivalent to  $\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2}$ . Press R/S to continue.

The calculator briefly displays RUNNING, then briefly displays NEXT ROOT, then stops and shows the next root calculated as a complex number. The display will show (if in rectangular mode):

$$C= \\ -5.1034\text{E-}12 \text{ i } 1.0000$$

This is equivalent to  $i$ . Press R/S to continue.

The calculator briefly displays RUNNING, then briefly displays NEXT ROOT, then stops and shows the next root calculated as a complex number. The display will show (if in rectangular mode):

$$C= \\ -0.7071 \text{ i } 0.7071$$

This is equivalent to  $-\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2}$ . Press R/S to continue.

The calculator briefly displays RUNNING, then briefly displays NEXT ROOT, then stops and shows the next root calculated as a complex number. The display will show (if in rectangular mode):

$$C= \\ -5.1034\text{E-}12 \text{ i } 1.0000$$

This is equivalent to  $i$ . Press R/S to continue.

The calculator briefly displays RUNNING, then briefly displays NEXT ROOT, then stops and shows the next root calculated as a complex number. The display will show (if in rectangular mode):

$$C= \\ -1.000 \text{ i } -1.0207\text{E-}11$$

This is equivalent to  $-1$ . Press R/S to continue.

The calculator briefly displays RUNNING, then briefly displays NEXT ROOT, then stops and shows the next root calculated as a complex number. The display will show (if in rectangular mode):

$$C= \\ -0.7071 \text{ i } -0.7071$$

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This is equivalent to  $-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$ . Press R/S to continue.

The calculator briefly displays RUNNING, then briefly displays NEXT ROOT, then stops and shows the next root calculated as a complex number. The display will show (if in rectangular mode):

$$C= \\ 1.5310E-11 \mathbf{i} -1.0000$$

This is equivalent to  $-i$ . Press R/S to continue.


The calculator briefly displays RUNNING, then briefly displays NEXT ROOT, then stops and shows the next root calculated as a complex number. The display will show (if in rectangular mode):

$$C= \\ 0.7071 \mathbf{i} -0.7071$$


This is equivalent to  $\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$ . Press R/S to continue.

The calculator displays PROGRAM END and finishes, resetting the Flag 10 and the angular mode.

**Complex Number Display**

Complex numbers in the HP-35s can be displayed in two ways in RPN mode. These are the rectangular mode and polar mode. Which display mode is set depends upon the display setting. To see complex numbers in rectangular form, press  **DISPLAY**, then 9. The  $x i y$  mode shows numbers as: 25.3456  $i$  -54.3210

The first part is the real part of the number, and the  $i$  separates this from the imaginary part, which is the second part. So the above number would be read as 25.3456  $- i$ 54.3210.

The other display mode, polar, is selected by pressing  **DISPLAY**, then selecting option 10 (scroll up or down to find it). This is shown as  $r \theta a$ , and shows first the distance, then the angle of this direction as an angle in decimal degrees counter-clockwise from the  $\Re$  (real) axis (horizontal axis to the right). If the calculator is in radians mode, the angle is displayed in radians (similarly in grads mode).

The same number as above would be shown as: 59.943060  $\theta$  -64.986744

where the 59.943060 is the distance, and the -60.986744 is the angle in decimal degrees.

Complex numbers may be entered in either mode, but are displayed in the selected mode. The mode can be set or changed at any time, to suit the user's needs.

**Roots of Complex Numbers****Sample Computations**

1. Solve for the 8th root of 1. See the solutions in the Running the Program Section.

2. Solve for the 4th roots of 1.

Solutions: 1st Root      1 (1 i 0)  
               2nd Root      i (0 i 1)  
               3rd Root      -1 (-1 i 0)  
               4th Root      -i (0 i -1)

3. Solve for the 3rd roots of  $3.5 + i 2.6$

Solutions: 1st Root      1.5968 i 0.3453  
               2nd Root      -1.0974 i 1.2102  
               3rd Root      -0.4993 i -1.5555

4. Solve for the 3rd roots of  $3.5 \theta 55$  (equivalent to  $2.0075 i 2.8670$ )

Solutions: 1st Root      1.4412 i 0.4776            or      1.5183  $\theta$  0.3200  
               2nd Root      -1.1342 i 1.0094            or      1.5183  $\theta$  2.4144  
               3rd Root      -0.3070 i -1.4869            or      1.5183  $\theta$  -1.7744

In the first case, the calculator is set to rectangular complex display, and in the second case, it is set to polar complex display. Note that the angles (the value after the  $\theta$ ) are in radians.

**Storage Registers Used**

- A**      Angular value in the complex number  
**B**      Value of  $\sqrt[n]{r}$  (i.e., the  $n$ th root of  $r$ )  
**C**      The first root of the complex number,  $z$ , in complex number format  
**D**      Subsequent roots of the complex number,  $z$ , in complex number format  
**L**      Loop counter, decremented each time through the loop  
**N**      The value  $n$ , that being the number of roots to be taken  
**R**      The absolute value (or length) of the complex number (or vector),  $z$ , i.e.,  $|z|$   
**Z**      The complex number,  $z$ , whose  $n$  roots are required

Statistical Registers:      not used

**Labels Used**

Label **R**      Length = 284      Checksum = 0467

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry.