Plane Triangle Solutions

Programmer: Dr. Bill Hazelton

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Line	Instruction	Display	Program Entry Instructions
T001	LBL T		LBL T
T002	CLSTK		CLEAR 5
T003	FS? 10		FLAGS SF? .0
T004	GTO T008		
T005	SF 1		FLAGS SF 1
T006	SF 10		FLAGS SF .0
T007	GTO T009		
T008	CF 1		FLAGS CF 1
T009	TRIANGLE SOLNS		EQN RCL T, RCL R, etc., ENTER to end
T010	PSE		
T011	0		
T012	STO T		
T013	3 SIDES		EQN 3 <SPACE> RCL S etc.
T014	PSE		
T015	INPUT T		
T016	RCL T		
T017	$x > 0$?		x?0 4
T018	GTO T044		
T019	2 SIDES INC θ		EQN 2 <SPACE> RCL S etc.
T020	PSE		
T021	INPUT T		
T022	RCL T		
T023	$x > 0$?		x?0 4
T024	GTO T120		
T025	2 SIDE N–INC θ		EQN 2 <SPACE> RCL S, etc.
T026	PSE		
T027	INPUT T		
T028	RCL T		
T029	$x > 0$?		x?0 4
T030	GTO T286		
T031	2 θ INC SIDE		EQN 2 <SPACE> θ RCL I, etc.
T032	PSE		
T033	INPUT T		
T034	RCL T		
T035	$x > 0$?		x?0 4
T036	GTO T180		
T037	2 θ N–INC SIDE		EQN 2 <SPACE> θ RCL I, etc.
T038	PSE		
T039	INPUT T		

Plane Triangle Solutions

Line	Instruction
T040	RCL T
T041	$x > 0$?
T042	GTO T233
T043	GTO T011
T044	ENTER SIDE 1
T045	PSE
T046	INPUT S
T047	RCL S
T048	STO A
T049	ENTER SIDE 2
T050	PSE
T051	INPUT S
T052	RCL S
T053	STO B
T054	ENTER SIDE 3
T055	PSE
T056	INPUT S
T057	RCL S
T058	STO C
T059	x^2
T060	RCL B
T061	x^2
T062	+
T063	RCL A
T064	x^2
T065	–
T066	RCL÷ B
T067	RCL÷ C
T068	2
T069	÷
T070	ACOS
T071	→HMS
T072	ANGLE 1 =
T073	PSE
T074	STOP
T075	RCL C
T076	x^2
T077	RCL A
T078	x^2
T079	+
T080	RCL B
T081	x^2
T082	–
T083	RCL÷ C
T084	RCL÷ A

Line	Instruction
T085	2
T086	÷
T087	ACOS
T088	→HMS
T089	ANGLE 2 =
T090	PSE
T091	STOP
T092	RCL B
T093	x^2
T094	RCL A
T095	x^2
T096	+
T097	RCL C
T098	x^2
T099	–
T100	RCL÷ A
T101	RCL÷ B
T102	2
T103	÷
T104	ACOS
T105	STO D
T106	→HMS
T107	ANGLE 3 =
T108	PSE
T109	STOP
T110	RCL D
T111	SIN
T112	2
T113	÷
T114	RCL× A
T115	RCL× B
T116	AREA =
T117	PSE
T118	STOP
T119	GTO T387
T120	ENTER SIDE 1
T121	PSE
T122	INPUT S
T123	RCL S
T124	STO A
T125	ENTER SIDE 2
T126	PSE
T127	INPUT S
T128	RCL S
T129	STO B

Line	Instruction
T130	ENTER ANGLE 3
T131	PSE
T132	INPUT Q
T133	RCL Q
T134	HMS→
T135	STO C
T136	COS
T137	RCL× A
T138	RCL× B
T139	2
T140	×
T141	RCL A
T142	x^2
T143	x < > y
T144	–
T145	RCL B
T146	x^2
T147	+
T148	√x
T149	STO D
T150	SIDE 3 =
T151	PSE
T152	VIEW D
T153	RCL C
T154	SIN
T155	RCL÷ D
T156	STO D
T157	RCL× A
T158	ASIN
T159	→HMS
T160	ANGLE 1 =
T161	PSE
T162	STOP
T163	RCL D
T164	RCL× B
T165	ASIN
T166	→HMS
T167	ANGLE 2 =
T168	PSE
T169	STOP
T170	RCL C
T171	SIN
T172	RCL× A
T173	RCL× B
T174	2

Plane Triangle Solutions

Line	Instruction
T175	÷
T176	AREA =
T177	PSE
T178	STOP
T179	GTO T387
T180	ENTER ANGLE 1
T181	PSE
T182	INPUT Q
T183	RCL Q
T184	HMS→
T185	STO A
T186	ENTER ANGLE 2
T187	PSE
T188	INPUT Q
T189	RCL Q
T190	HMS→
T191	STO B
T192	ENTER SIDE 3
T193	PSE
T194	INPUT S
T195	RCL S
T196	STO C
T197	180
T198	RCL– A
T199	RCL– B
T200	STO D
T201	→HMS
T202	ANGLE 3 =
T203	PSE
T204	STOP
T205	RCL C
T206	RCL D
T207	SIN
T208	STO E
T209	÷
T210	STO F
T211	RCL A
T212	SIN
T213	×
T214	STO× E
T215	SIDE 1 =
T216	PSE
T217	STOP
T218	RCL F
T219	RCL B

Line	Instruction
T220	SIN
T221	×
T222	STO× E
T223	SIDE 2 =
T224	PSE
T225	STOP
T226	RCL E
T227	2
T228	÷
T229	AREA =
T230	PSE
T231	STOP
T232	GTO T387
T233	ENTER ANGLE 1
T234	PSE
T235	INPUT Q
T236	RCL Q
T237	HMS→
T238	STO A
T239	ENTER ANGLE 3
T240	PSE
T241	INPUT Q
T242	RCL Q
T243	HMS→
T244	STO B
T245	ENTER SIDE 3
T246	PSE
T247	INPUT S
T248	RCL S
T249	STO C
T250	RCL B
T251	SIN
T252	÷
T253	STO D
T254	RCL A
T255	SIN
T256	×
T257	SIDE 1 =
T258	PSE
T259	STOP
T260	180
T261	RCL– A
T262	RCL– B
T263	STO E
T264	→HMS

Line	Instruction
T265	ANGLE 2 =
T266	PSE
T267	STOP
T268	RCL E
T269	SIN
T270	RCL× D
T271	STO E
T272	SIDE 2 =
T273	PSE
T274	STOP
T275	RCL E
T276	RCL× C
T277	RCL A
T278	SIN
T279	2
T280	÷
T281	×
T282	AREA =
T283	PSE
T284	STOP
T285	GTO T387
T286	ENTER SIDE 1
T287	PSE
T288	INPUT S
T289	RCL S
T290	STO A
T291	ENTER SIDE 2
T292	PSE
T293	INPUT S
T294	RCL S
T295	STO B
T296	ENTER ANGLE 1
T297	PSE
T298	INPUT Q
T299	RCL Q
T300	HMS→
T301	STO C
T302	SIN
T303	RCL÷ A
T304	STO D
T305	RCL× B
T306	ASIN
T307	STO E
T308	180
T309	x < > y

Plane Triangle Solutions

T310	–
T311	RCL– C
T312	STO F
T313	SIN
T314	RCL× A
T315	RCL C
T316	SIN
T317	÷
T318	STO G
T319	RCL F
T320	SIN
T321	RCL× A
T322	RCL× B
T323	2
T324	÷
T325	STO H
T326	SOLUTION 1
T327	PSE
T328	ANGLE 2 =
T329	PSE
T330	RCL E
T331	→HMS
T332	STOP
T333	ANGLE 3 =
T334	PSE
T335	RCL F
T336	→HMS
T337	STOP
T338	SIDE 3 =
T339	PSE
T340	RCL G
T341	STOP
T342	AREA =
T343	PSE
T344	RCL H
T345	STOP
T346	180
T347	RCL– E
T348	STO E
T349	180
T350	x < > y
T351	–
T352	RCL– C
T353	STO F
T354	SIN
T355	RCL× A

T356	RCL C
T357	SIN
T358	÷
T359	STO G
T360	RCL F
T361	SIN
T362	RCL× A
T363	RCL× B
T364	2
T365	÷
T366	STO H
T367	SOLUTION 2
T368	PSE
T369	ANGLE 2 =
T370	PSE
T371	RCL E
T372	→HMS
T373	STOP
T374	ANGLE 3 =
T375	PSE
T376	RCL F
T377	→HMS
T378	STOP
T379	SIDE 3 =
T380	PSE
T381	RCL G
T382	STOP
T383	AREA =
T384	PSE
T385	RCL H
T386	STOP
T387	FS? 1
T388	CF 10
T389	RTN

Plane Triangle Solutions

Notes

(1) Program for solving a plane triangle's unspecified angles, side lengths and area, given three inputs (angles and side lengths) that include at least one side length.

(2) Angles are entered and displayed in HP notation, i.e., DDD.MMSS.

(3) Whatever linear units are used (and they should be the same for all three sides, of course), the area will be presented in those units squared. That is, if the lengths are in feet, the area is in square feet; if the lengths are in meters, the area is in square meters; if the lengths are in cubits, the area is in square cubits.

(4) The purpose of the EQN entries in the program is to provide a prompt ahead of input or output. The program sets flag 10 to display rather than evaluate equations. Letters of the message must be entered with the RCL key, i.e., to enter HI, press RCL H then RCL I. Spaces can be entered with the SPACE key combination. Digits can be keyed in directly.

(5) The two angles and included side problem is essentially a surveying 'intersection' problem, and can also be interpreted as the 'two missing sides' problem.

(6) In the '2 sides and an angle not between them' problem, there are two possible solutions. Each solution is presented separately. Note that if one of the solutions is not physically possible, the program may return an error.

(7) The program has all the possible solutions coded within it, so the user is first presented with a set of choices, one at a time, to select the appropriate solution. When a selection is made, the calculator then moves to the correct part of the program and proceeds with a stand-alone calculation. At the end of the operation, the program returns Flag 10 to its original setting, before returning to the calling point.

Label Used

Label **T** Length = 1664 Checksum = EB95

Use the length (LN=) and Checksum (CK=) values to check if program was entered correctly. Use the sample computation to check proper operation after entry. Length and checksum values are based on single spaces between words, numbers and equal signs in prompts.

Storage Registers Used

A Input 1

B Input 2

C Input 3

D Intermediate result storage

E Intermediate result storage

F Intermediate result storage

G Intermediate result storage

Plane Triangle Solutions

H Intermediate result storage

Q Temporary storage for entered angles

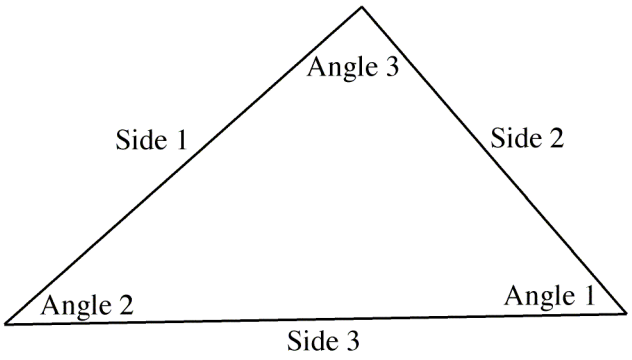
S Temporary storage of entered side length

T Temporary storage of choice value for initial selection of sub-program.

Theory

This program accepts the three components of a triangle, then uses the sine rule, cosine rule and angle sum conditions to compute the three remain components. The area is computed using the length of two sides and half the sine of the angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number.



The resulting angles are presented in HP notation. Plane surveying assumptions apply. The program uses no error checking on entered data or results. It is a good move to check that the angles all sum to 180°.

Running the Program

Press XEQ T and press ENTER.

The calculator displays TRIANGLE SOLNS briefly.

The calculator then moves into a large loop to determine the particular type of triangle that needs to be solved. It displays the options for solution in turn, and offers a prompt for selection. This loop continues until a choice is made.

A. The calculator displays 3 SIDES briefly.

If the three side lengths of the triangle are known, then this is the solution required. If not, it isn't.

The calculator displays a prompt and default value, and waits for input:

T?
0.00000

If the three known sides solution is required, key in 1 and press R/S. If not, key in zero and press R/S, or just press R/S, as zero is the default.

If 1 was entered, the program jumps to the 3 SIDES solution, discussed below at **B**.

Plane Triangle Solutions

If a zero was entered, or just R/S pressed, the calculator briefly displays:

2 SIDES INC θ

then the prompt for input:

T?
0.00000

This is the point for the solution where two side lengths are known, together with the angle between the two known sides. If this is desired, key in 1 and press R/S. if not, just press R/S to go to the next choice.

If 1 was entered, the program jumps to the 2 SIDES INC θ solution, discussed below at **C**.

If a zero was entered, or just R/S pressed, the calculator briefly displays:

2 SIDES N–INC θ

then the prompt for input:

T?
0.00000

This is the point for the solution where two side lengths are known, together with an angle that is not between the two known sides. If this is desired, key in 1 and press R/S. if not, just press R/S to go to the next choice.

If 1 was entered, the program jumps to the 2 SIDES N–INC θ solution, discussed below at **D**.

If a zero was entered, or just R/S pressed, the calculator briefly displays:

2 θ INC SIDE

then the prompt for input:

T?
0.00000

This is the point for the solution where two angles are known, together with the side length between the two angles. If this is desired, key in 1 and press R/S. if not, just press R/S to go to the next choice.

If 1 was entered, the program jumps to the 2 θ INC SIDE solution, discussed below at **E**.

If a zero was entered, or just R/S pressed, the calculator briefly displays:

2 θ N–INC SIDE

then the prompt for input:

T?
0.00000

This is the point for the solution where two angles are known, together with a side length that is not directly between the two angles. If this is desired, key in 1 and press R/S, if not, just press R/S to go to the next choice.

If 1 was entered, the program jumps to the 2 θ N–INC SIDE solution, discussed below at **F**.

If a zero was entered, or just R/S pressed, the calculator returns to the first option, 3 SIDES, as it has run through all of the possibilities with plain triangles. So the calculator returns to point **A**, above.

B. 3 SIDES solution

If the 3 SIDES solution was chosen the calculator jumps to this point, and the proceeds as follows.

Screen shows ENTER SIDE 1 briefly, then prompts with S?

Enter the length of side 1 and press R/S.

Screen shows ENTER SIDE 2 briefly, then prompts with S?

Enter the length of side 2 and press R/S.

Screen shows ENTER SIDE 3 briefly, then prompts with S?

Enter the length of side 3 and press R/S.

Screen shows ANGLE 1 = briefly, then shows Angle 1 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 2 = briefly, then shows Angle 2 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 3 = briefly, then shows Angle 3 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to end program. This sets Flag 10 to its original value, as it was set at the start of the program.

Sample Computations

	Triangle 1	**Triangle 2**
Inputs:	Side Length 1 = 100.000	Side Length 1 = 10.000
	Side Length 2 = 100.000	Side Length 2 = 10.000
	Side Length 3 = 100.000	Side Length 3 = 18.000
Results:	Angle 1 = 60° 00' 00"	Angle 1 = 25° 50' 31"
	Angle 2 = 60° 00' 00"	Angle 2 = 25° 50' 31"
	Angle 3 = 60° 00' 00"	Angle 3 = 128° 18' 58"
	Area = 4,330.127	Area = 39.230
	Check angle sum = 180° 00' 00"	Check = 180° 00' 00"

Plane Triangle Solutions

C. 2 SIDES INC θ solution

If the 2 SIDES INC θ solution was chosen, the program jumps to this point, and proceeds as follows.

Screen shows ENTER SIDE 1 briefly, then prompts with S?

Enter the length of side 1 and press R/S.

Screen shows ENTER SIDE 2 briefly, then prompts with S?

Enter the length of side 2 and press R/S.

Screen shows ENTER ANGLE 3 briefly, then prompts with Q?

Enter angle 3 in HP notation and press R/S.

Screen shows SIDE 1 = briefly, then shows Side 1 in the lower (X) register, with D= above.

Press R/S to continue.

Screen shows ANGLE 1 = briefly, then shows Angle 1 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 2 = briefly, then shows Angle 2 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to end program. This resets Flag 10 to its previous value, as it was set at the start of the program.

Sample Computations

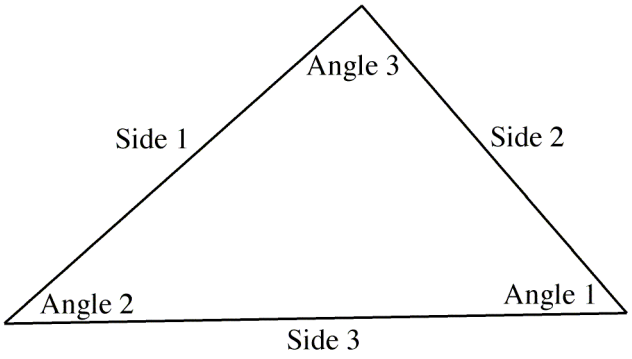
	Triangle 1	**Triangle 2**
Inputs:	Side Length 1 = 100.000	Side Length 1 = 10.000
	Side Length 2 = 100.000	Side Length 2 = 10.000
	Angle 3 = 60° 00' 00"	Angle 3 = 128° 18' 58"
Results:	Side 3 = 100.000	Side 3 = 18.000
	Angle 1 = 60° 00' 00"	Angle 1 = 25° 50' 31"
	Angle 2 = 60° 00' 00"	Angle 2 = 25° 50' 31"
	Area = 4,330.127	Area = 39.230
	Check angle sum = 180° 00' 00"	Check = 180° 00' 00"

Plane Triangle Solutions

D. 2 SIDES N–INC θ Solution

This part of the program accepts two side lengths of the triangle and an angle not between them, then uses the sine rule to compute one of the other angles. The third angle is computed by subtracting the sum of the other angles from 180°. The remaining side is computed using the sine rule. The area is computed using the length of two sides and half the sine of the angle between them.

The numbering scheme for the sides and angles is as shown in the diagram. Sides are opposite the angle with the same number. In this case, Side 1 and Side 2 are known, along with Angle 1.



There are two possible solutions, depending upon the solutions to Angle 2. Because it is determined using the sine rule, and arcsin(x) can have two values, there is one solution where Angle 2 lies between 0° and 90°, and a second where Angle 2 lies between 90° and 180°. Both these solutions are computed. The results are presented in two groups, with suitable text prompts.

Solution 1 is based on Angle 2 being less than 90°. Solution 2 is based on Angle 2 being greater than 90°.

When the 2 SIDES N–INC θ solution is chosen, the calculator jumps to this point, and proceeds as follows.

Screen shows ENTER SIDE 1 briefly, then prompts with S?

Enter the length of side 1 and press R/S.

Screen shows ENTER SIDE 2 briefly, then prompts with S?

Enter the length of side 2 and press R/S.

Screen shows ENTER ANGLE 1 briefly, then prompts with Q?

Enter angle 1 in HP notation and press R/S.

Screen shows SOLUTION 1 briefly.

Screen shows ANGLE 2 = briefly, then shows Angle 2 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 3 = briefly, then shows Angle 3 in HP notation in the lower (X) register.

Press R/S to continue.

Plane Triangle Solutions

Screen shows SIDE 3 = briefly, then shows Side 3 in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to continue.

Screen shows SOLUTION 2 briefly.

Screen shows ANGLE 2 = briefly, then shows Angle 2 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 3 = briefly, then shows Angle 3 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows SIDE 3 = briefly, then shows Side 3 in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to end program. This resets Flag 10 to its original value, as it was set at the start of the program.

Sample Computations

	Triangle 1	Triangle 2
Inputs:	Side Length 1 = 100.000	Side Length 1 = 10.000
	Side Length 2 = 100.000	Side Length 2 = 10.000
	Angle 1 = 60° 00' 00"	Angle 1 = 25° 50' 31"
Results:	**Solution 1**	
	Angle 2 = 60° 00' 00"	Angle 2 = 25° 50' 31"
	Angle 3 = 60° 00' 00"	Angle 3 = 128° 18' 58"
	Side Length 3 = 100.000	Side Length 3 = 18.000
	Area = 4,330.127	Area = 39.230
	[Check angle sum = 180° 00' 00"	Check = 180° 00' 00"]
	Solution 2	
	Angle 2 = 120° 00' 00"	Angle 2 = 154° 09' 29"
	Angle 3 = 0° 00' 00"	Angle 3 = 0° 00' 00" (very small)
	Side Length 3 = 0.000	Side Length 3 = 0.000
	Area = 0.000	Area = 0.000
	Check angle sum = 180° 00' 00"	Check = 180° 00' 00"

Plane Triangle Solutions

Clearly, neither of the results for Solution 2 are particularly good solutions (despite being mathematically valid and correct), because of the zero angles involved. This, as well as negative angles, is one way to spot an unsuitable solution. However, it is possible to have two perfectly reasonable solutions, in which case you will need to look beyond the given data to decide which is the required solution.

E. 2 θ INC SIDE Solution

When the 2 θ INC SIDE solution is chosen, the calculator jumps to this point, and proceeds as follows.

Screen shows ENTER ANGLE 1 briefly, then prompts with Q?

Enter angle 1 in HP notation and press R/S.

Screen shows ENTER ANGLE 3 briefly, then prompts with Q?

Enter angle 3 in HP notation and press R/S.

Screen shows ENTER SIDE 3 briefly, then prompts with S?

Enter the length of side 3 and press R/S.

Screen shows SIDE 1 = briefly, then shows Side 1 in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 1 = briefly, then shows Angle 1 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows SIDE 2 = briefly, then shows Side 2 in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to end program. This sets Flag 10 to its original value, as it was set at the start of the program.

Sample Computations

	Triangle 1	Triangle 2
Inputs:	Angle 1 = 60° 00' 00"	Angle 1 = 25° 50' 31"
	Angle 2 = 60° 00' 00"	Angle 2 = 25° 50' 31"
	Side Length 3 = 100.000	Side Length 3 = 18.000
Results:	Angle 3 = 60° 00' 00"	Angle 3 = 128° 18' 58"
	Side 1 = 100.000	Side 1 = 10.000
	Side 2 = 100.000	Side 2 = 10.000
	Area = 4,330.127	Area = 39.230
	Check angle sum = 180° 00' 00"	Check = 180° 00' 00"

Plane Triangle Solutions

F. 2 θ N–INC SIDE Solution

When the 2 θ N–INC SIDE solution is chosen, the calculator jumps to this point, and proceeds as follows.

Screen shows ENTER ANGLE 1 briefly, then prompts with Q?

Enter angle 1 in HP notation and press R/S.

Screen shows ENTER ANGLE 3 briefly, then prompts with Q?

Enter angle 3 in HP notation and press R/S.

Screen shows ENTER SIDE 3 briefly, then prompts with S?

Enter the length of side 3 and press R/S.

Screen shows SIDE 1 = briefly, then shows Side 1 in the lower (X) register.

Press R/S to continue.

Screen shows ANGLE 1 = briefly, then shows Angle 1 in HP notation in the lower (X) register.

Press R/S to continue.

Screen shows SIDE 2 = briefly, then shows Side 2 in the lower (X) register.

Press R/S to continue.

Screen shows AREA = briefly, then shows the area in the lower (X) register.

Press R/S to end program. This resets Flag 10 to its original value, as it was set at the start of the program.

Sample Computations

	Triangle 1	Triangle 2
Inputs:	Angle 1 = 60° 00' 00"	Angle 1 = 25° 50' 31"
	Angle 3 = 60° 00' 00"	Angle 3 = 128° 18' 58"
	Side Length 3 = 100.000	Side Length 3 = 18.000
Results:	Side 1 = 100.000	Side 1 = 10.000
	Angle 2 = 60° 00' 00"	Angle 2 = 25° 50' 31"
	Side 2 = 100.000	Side 2 = 10.000
	Area = 4,330.127	Area = 39.230
	Check angle sum = 180° 00' 00"	Check = 180° 00' 00"